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Deep Learning for the Closure and Solution of Partial Differential Equations

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Although the exact physics equations for an application may be available, numerically solving these equations can be computationally intractable. The exact physics typically involve complex phenomena at small scales, which can require an infeasibly large computational grid to accurately resolve. An example is turbulence, which is relevant to modeling airplanes, biomedical technology, and power generation. Large-eddy simulation (LES) is a reduced-order PDE model for the low frequencies of the Navier-Stokes equations for turbulent flows. By modeling only the low frequencies, the LES equations can be solved at a low computational cost on a coarse grid. However, the LES equations introduce an unclosed term which must be modeled. We develop a deep learning closure model for LES. The "deep learning LES model" is calibrated to high-fidelity data. Training uses adjoint PDEs to optimize over the full nonlinearity of the PDE model. The approach is implemented for isotropic turbulence, turbulent jet flows, and turbulent wakes. In these LES applications, the PDE model itself is determined by a neural network and then solved using traditional numerical methods. In the last part of the presentation, directly solving PDEs using neural networks and gradient descent-type algorithms will be discussed. For a class of linear PDEs, we prove that asymptotically the PDE residual of the neural network weakly converges to zero.