

CRUNCH Seminars at Brown, Division of Applied Mathematics

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Recent Developments in the Mathematics of Neural Nets

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A profound mathematical mystery of our times is to be able to explain the phenomenon of training neural nets i.e "deep-learning". The dramatic progress of this approach in the last decade has gotten us the closest we have ever been to achieving "artificial intelligence". But trying to reason about these successes - for even the simplest of nets - immediately lands us into a plethora of extremely challenging mathematical questions, typically about discrete stochastic processes. In this talk we will describe the various themes of our work in provable deep-learning.

We will start with a brief introduction to neural nets and then see glimpses of our initial work on understanding neural functions, loss functions for autoencoders, and algorithms for exact neural training. Next, we will explain our recent result about how under mild distributional conditions we can construct an iterative algorithm that can be guaranteed to train a ReLU gate in the realizable setting in linear time while also keeping track of mini-batching - and its provable graceful degradation of performance under a data-poisoning attack. We will show via experiments the intriguing property that our algorithm very closely mimics the behaviour of Stochastic Gradient Descent (S.G.D.), for which similar convergence guarantees are still unknown.

Lastly, we will review this very new concept of "local elasticity" of a learning process and demonstrate how it appears to reveal certain universal phase transitions during neural training. Then we will introduce a mathematical model which reproduces some of these key properties in a semi-analytic way. We will end by delineating various exciting future research programs in this theme of macroscopic phenomenology with neural nets.