

CRUNCH Seminars at Brown, Division of Applied Mathematics

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SIAM-ACM Prize in Computational Science and Engineering

**Lecture: DeepOnet: Learning Linear, Nonlinear and
Multiscale Operators Using Deep Neural Networks Based on
the Universal Approximation Theorem of Operators**

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It is widely known that neural networks (NNs) are universal approximators of continuous functions. However, a less known but powerful result is that a NN with a single hidden layer can accurately approximate any nonlinear continuous operator. This universal approximation theorem of operators is suggestive of the structure and potential of NNs in learning continuous operators or complex systems from streams of scattered data. We extend this theorem to deep neural networks (DNNs). We design a new network with small generalization error, the deep operator network (DeepONet), consisting of a DNN for encoding the discrete input function space (branch net) and another DNN for encoding the domain of the output functions (trunk net). We demonstrate that DeepONet can learn various explicit operators, such as integrals and fractional Laplacians, as well as implicit operators that represent deterministic and stochastic differential equations. We study different formulations of the input function space and its effect on the generalization error for realistic applications across different application domains.