

Saving and growth: a reinterpretation*

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Abstract

We examine the relationship between income growth and saving using both cross-country and household data. At the aggregate level, we find that growth Granger causes saving, but saving does not Granger cause growth. Using household data, we find that households with predictably higher income growth save more than households with predictably low growth. We argue that standard permanent income models of consumption cannot explain these findings, but a model of consumption with habit formation may. The positive effect of growth on saving implies that previous estimates of the effect of saving on growth may be overstated.

1 Introduction

In this paper we reexamine the relationship between the rate of income growth and the saving rate. The recent literature on economic growth has

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found that countries with higher saving or investment rates have tended to have higher growth rates (see Levine and Renelt, 1992, for a review). This finding has been interpreted as being consistent with either the traditional Solow (1956) growth model, in which higher saving leads to higher level of income per capita in steady state (and thus to higher growth rates on the transition path), or with the “new growth models” of Romer (1987) and others in which higher saving leads to a permanently higher rate of growth.

An obvious problem in interpreting the results of a regression of growth on saving is that the level of growth may itself affect the saving rate. Modigliani (1970) showed many years ago that a very simple version of the life-cycle model can predict that high growth causes high saving, and he found empirical support for the theoretical prediction using cross-country data. More recently, Baumol, Blackman, and Wolfe (1991), Deaton and Paxson (1992), and Bosworth (1993) have also provided evidence that faster growth may raise saving. This paper explores the empirical relationship between saving and growth using both aggregate and household data, and from a variety of different perspectives. We consistently find evidence that higher income growth produces greater saving. We then argue that our results are not consistent with a strict interpretation of the usual models of consumption and growth, and we consider several alternatives.

The rest of this paper is organized as follows. In Section 2, we begin by confirming Modigliani’s empirical finding that countries which have high growth also have high saving. We then examine the predictions of the neo-classical growth model for the relation between saving and growth. We show that in that model, exogenous increases in growth make subsequent saving fall, while exogenous increases in saving make subsequent growth rise. We then examine the empirical links between saving and growth within individual countries over time. We find that increases in growth are followed by increases in saving – a result that is not consistent with either of the theoretical predictions from the neoclassical model.

In Section 3, we turn to household-level data which are not plagued by the general equilibrium effects that cloud aggregate tests. We use three different data sets, and use both the saving rate and the wealth/income ratio as our dependent variables. Using this data we also find that saving is positively correlated with income growth.

In Section 4 we discuss the implications of these results for the theory of consumption. In looking for explanations of the positive effect of growth on saving in both household and aggregate data, we are guided by Occam’s Razor: a single explanation that encompasses both phenomena is preferable to separate explanations for each of the two. We discuss the ability of consumption models incorporating uncertainty and liquidity constraints to explain our findings, and conclude that they are not sufficient to do the

job. We then argue that our results might stem from habit-formation behavior in consumption, although it appears that the degree of habit persistence required to explain our results is rather high. Section 5 concludes.

2 The relation between growth and saving at the aggregate level

2.1 *Facts on the long-term relationship between saving and growth*

We begin by examining the empirical relationship between growth rates and saving rates in cross-country data. We use two samples of countries in our work. We started with the Summers and Heston (1991) Mark 5 data set, and then excluded all countries whose data received a grade of lower than “C-”. We further excluded communist countries, countries whose economies were dominated by oil production, and countries with 1985 populations of less than one million. The remaining sample consisted of 64 countries; we call this our “full” sample. Our second sample is the 22 members of the OECD with 1985 populations greater than one million.

Table 1 presents simple cross-sectional regressions of national saving rates¹ on growth, both including and excluding the initial log of output per capita from the right-hand side. These regressions resemble the “growth regressions” presented by Barro (1991), among many others, except that we have made saving the dependent variable and growth an independent variable rather than the reverse. Of course, putting saving on the left-hand side does not prove that causation runs from growth to saving any more than putting growth on the left-hand side proves that causation runs from saving to growth.²

When growth alone is the right-hand side variable, it enters significantly in the full sample and with a t-statistic of 1.65 in the OECD sample. The correlations between average growth and average saving are .35 for the OECD and .26 for the full sample. When the log of income per capita in 1960 is partialled out, there is a very significant relation between growth and saving in the OECD sample and a borderline significant relation in the full sample.

In Table 2, we look at the relation between growth and saving within countries over time by running panel regressions using the Summers and

¹The measure we examine is nominal national saving as a fraction of nominal national income. Unlike measures of investment, our measure is not affected by differences in the relative price of investment goods examined by DeLong and Summers (1991). An exact description of our measure of saving can be found in the Data Appendix.

²King and Levine (1994) run regressions similar to ours, with growth rates on the right-hand side and investment as the dependent variable. Their results are consistent with a model in which growth rates differ exogenously across countries and in which investment acts to keep the capital/output ratio constant. They argue that this model, rather than causation running from investment to growth, may be the explanation for the observed correlation between investment and growth.

Table 1:
 Cross-Section Regressions
 Dependent Variable: Average Saving Rate 1960-87

	Sample:			
	OECD	OECD	Full	Full
constant	.168 (.034)	-.530 (.205)	.169 (.016)	-.063 (.068)
grow6087	1.84 (1.11)	4.73 (1.23)	1.19 (.59)	1.06 (.54)
ln(y60)		.0725 (.0211)		.0304 (.0088)
Observations	22	22	59	59
\bar{R}^2	.076	.400	.051	.205
Root MSE	.0462	.0372	.0586	.0537

Note: Standard errors in parentheses. grow6087 is the average annual growth of real per-capita output over the period 1960-87. ln(y60) is the log of real per-capita output in 1960. Saving is nominal national saving as a fraction of nominal national income – see Data Appendix.

Heston data. For each country we look at nonoverlapping five-year averages of growth and savings rates. We use data from 1958–1987, giving a maximum (if there are not missing years) of six observations for each country. By taking five-year averages we hope to avoid picking up business-cycle frequency relations between growth and saving. In all regressions we include a full set of country dummies on the right-hand side, and in addition we experiment with controlling for the log of initial income per capita during the period and allowing for a full set of time-period effects. In the full sample, the growth rate is always significant, while in the OECD sample the growth rate is significant as long as either year effects or the log of output is included.

Table 2:
Panel Regressions
Dependent Variable: s_t

Sample	OECD	OECD	OECD	OECD	Full	Full	Full	Full
g_t	.172 (.124)	.432 (.136)	.367 (.150)	.364 (.153)	.376 (.090)	.528 (.079)	.509 (.092)	.439 (.084)
$\ln(y)$.0287 (.0076)		-.0016 (.0194)		.0750 (.0072)		.1152 (.0138)
time effects			.000	.055			.000	.002
Observations	132	132	132	132	369	369	369	369
\bar{R}^2	.817	.837	.847	.846	.671	.757	.715	.768
Root MSE	.022	.0208	.020	.0202	.042	.0358	.0388	.0350

Note: Standard errors in parentheses. All regressions include a full set of country dummies. The number in the row marked “time effects” is the p -value from the test that the coefficients on the set of included time dummies are zero. s_t and g_t are average saving and growth rates over five-year periods.

Ours is by no means the first evidence suggesting a powerful link from income growth to saving. Modigliani (1970) found results similar to those in our Table 1 long ago. More recently, in a comprehensive study of the determinants of saving rates in the OECD countries in the period from the 1960s to the 1980s, Bosworth (1993) found that the growth rate of income was the most important determinant of saving. Looking over longer spans of time, Maddison (1992) also finds a positive relation between saving and growth. For the seven countries for which data are available for the period 1870–1913, the correlation between saving and growth rates is .58.³ For the six countries for which data is available for the period 1914–1950, the correlation

³Saving data for India are for the period 1890–1913.

is .67. Another important result that comes from Maddison's work is that Kuznets' (1946) finding that the saving rate in the United States had been relatively constant for the last century represents something of an outlier. Of the eleven countries for which Maddison presents long-time series on saving, the United States is the only one in which the saving rate does not show a significant increase over time. The United States is also the only country which experienced almost no increase in the growth rate of output over the 120-year period which Maddison examines.

2.2 *What does theory predict?*

As we remarked earlier, Modigliani (1970, and many others) has argued that the positive cross-country association of saving and income growth is evidence in favor of the life-cycle model of saving. Modigliani notes that if there were no productivity growth across generations, and no population growth, the saving of the young would exactly balance the dissaving of the old, and the net national saving rate would be zero. Because productivity growth makes the young richer than the old, the young will be saving more than the old are dissaving (assuming the saving *rate* of the young is the same as the rate at which the current old saved when they were young).

A peculiar feature of Modigliani's model, however, is that he assumes that the income growth rate for individual consumers is no higher in a high-growth economy than in a low-growth economy. Aggregate income growth is the result of increasing the *level* of the lifetime income profile for succeeding generations. In other words, in Modigliani's framework there would be no reason to expect that the growth rate of income for an individual Japanese worker over the last 40 years was any greater than the growth rate of income for a British worker of the same age.

Carroll and Summers (1991) muster a range of evidence against this description of the relationship between aggregate and individual income growth. They argue that a better description is that household income growth g_i is equal to aggregate income growth g plus adjustments for seniority, occupation, and other individual-specific factors, e_i .

If household income growth is given by $g_i = g + e_i$, an exogenous increase in aggregate growth g will make every household want to consume more and save less. As noted by Tobin (1967), under reasonable parameter values this effect typically outweighs Modigliani's aggregation effect so that the predicted correlation between aggregate income growth and saving becomes negative. Thus, even without augmenting the model with general equilibrium effects, the model's prediction about the correlation between aggregate saving and growth is ambiguous.

The life-cycle model produces much cleaner implications for the relation between growth and saving at the household level than at the aggregate level,

so we will postpone further discussion of that model until we have presented the household-level evidence. In the remainder of this section of the paper we will examine the standard neoclassical model of optimal growth, in which analysis of general equilibrium effects is at least somewhat manageable.

2.2.1 The relation between saving and growth in the neoclassical model. We consider a standard, closed-economy neoclassical model of optimal growth. Utility in each period is given by a constant relative risk-aversion utility function, and consumption is equal to the level that would be chosen by a social planner maximizing the discounted sum of future utility:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\rho}}{1-\rho} \quad (1)$$

where ρ is the coefficient of relative risk aversion, and β is the discount factor equal to $1/(1 + \theta)$, where θ is the discount rate. We assume that production is Cobb-Douglas with constant returns to scale, that labor is supplied inelastically, and that there is no population growth. We also assume labor-augmenting exogenous technological growth at rate λ . Output is thus

$$Y_t = AK_t^\alpha ((1 + \lambda)^t L_t)^{1-\alpha} \quad (2)$$

Capital accumulation is given by

$$K_{t+1} = Y_t - C_t + (1 - d)K_t \quad (3)$$

where d is the rate of depreciation.

In the steady state, the growth rate of income in this model is determined by the technological growth parameter λ and does not depend on the saving rate. In the short- and medium-run, however, there are several different channels through which saving and growth are related, and the sign and magnitude of the correlation between the two are theoretically ambiguous.

The most intuitive channel is the direct relation between saving, capital accumulation, and the level of income, embodied in equations (2) and (3). Given an initial level of capital and output, exogenously higher saving will lead to higher capital accumulation and so higher output growth in the short- to medium-run. This is the linkage from saving to growth examined in Mankiw, Romer, and Weil (1992), among many other papers, in which the long-term saving rate is treated as an independent variable. We will refer to this as the “mechanical link” from saving to growth. The length of time over which this link is important depends on the weight of capital in the production function. If there are constant returns to capital, as suggested by Romer (1987), then the effect of saving on growth lasts indefinitely. If capital is less important, the effect can be short-lived. Mankiw, Romer, and Weil

show that, taking the saving rate as exogenous, the half-life of deviations of output from the steady-state level is inversely proportional to $(1 - \alpha)$.

In a model in which consumption is determined by forward-looking consumers, however, a powerful link between growth and consumption runs in the opposite direction. If growth is exogenously higher, then, *ceteris paribus*, forward-looking consumers will feel wealthier and will spend more and save less. We will call this the “human-wealth link” from expected growth to saving.

A third set of links arises indirectly as a result of the relationship between interest rates and consumption. If countries have identical preferences and technologies but differ in their initial capital endowments, then poor countries should have both high growth rates and high interest rates. Traditional consumption analysis finds that higher interest rates affect consumption through the substitution effect (which raises saving), the income effect (which lowers saving), and the human-wealth effect (which raises saving). The net sign of these effects depends on parameter values, but Summers (1981) has argued that for plausible parameter values the model implies that the interest elasticity of saving should be strongly positive. We will refer to the net effect of higher interest rates on consumption as the “interest-rate effect” on saving.⁴

The parameters of the model determine the strength of the various linkages. In particular, ρ , the inverse of the intertemporal elasticity of substitution, governs the strength of the interest-rate effect. If ρ is low (the intertemporal elasticity is high), then consumers will be more willing to postpone consumption today in order to enjoy more consumption tomorrow, so the interest-rate effect on consumption will be large. In the experiments below, we consider values of ρ of one (log utility) and four. Log utility is often used in analyzing consumption models because it has convenient analytical properties. However, empirical evidence appears to indicate higher values of ρ , and our second choice of four lies at the low end of many empirical estimates.⁵

Another important parameter is α , the exponent on capital in the production function, which determines the extent to which a lower capital stock will raise the rate of return on capital, which in turn raises the rates of saving and growth. King and Rebelo (1993) show that for low values of α , the implied interest rates when output is well below its steady-state level are quite high. Mankiw, Romer, and Weil (1992) argue that for an extended

⁴The analysis of the model presented here is for a closed economy. In the case of an economy open to a world capital market, the influences leading to a negative relation between income and saving would be stronger. In particular, while the human-wealth effect of future wages on consumption would still be present, neither the mechanical link from saving to growth nor the interest-rate effect would be operative in such an economy.

⁵Hall (1988) found a minimum estimated value of 5 for ρ using United States aggregate data; Mankiw and Zeldes (1991) found a minimum value of 6 using household data.

definition of capital that includes human capital, a value of α of two-thirds is appropriate. This is the value that we use in our simulations.

Because of the multiple and countervailing influences linking growth and saving, the relationship between the two that one should expect to observe in aggregate data is crucially dependent on the sources of variation across countries. We consider two experiments which demonstrate this point. The first is a shock to the discount rate, Θ . We assume that a country is in steady state with discount rate of 4%. In year zero, the discount rate is lowered to 3%. Although we do not think that people's discounting of future utility is really subject to abrupt exogenous changes, changing the discount rate can proxy for other changes that might affect countries. A plausible story about economic development, for instance, might hold that development can begin when a country's government realizes the long-term benefits of increased saving and embarks on a national program of saving and investment explicitly designed to achieve growth by exploiting the mechanical link between saving and growth. The simplest way to model such a shift might be as a change in the country's discount rate.⁶

Figure 1 shows the results of this experiment graphically. When $\rho = 1$, the saving rate increases by 7 percentage points immediately, and gradually declines toward a new permanent level that is approximately 5 points higher than before. The annual growth rate leaps up to about 2.8 percent and then declines relatively rapidly back toward its 2-percent equilibrium. When $\rho = 4$ (the bottom two panels), the saving rate increases by little more than 1 percentage point, but does not change significantly thereafter, while the growth rate of income jumps to $2\frac{1}{4}$ percent and then gradually declines back toward 2 percent. In both cases, then, the increase in saving is associated with a substantial and long-lasting subsequent increase in growth.

The second experiment we consider is farther outside the traditional growth literature: a change in the exogenous rate of technological progress, λ . Although we doubt that countries can be viewed as having permanent differences in their growth rates of technology, the growth experiences of a number of countries seem to be characterized by changes in broadly defined technology (including property rights, the degree and nature of government interference in markets, and restrictions on trade). A salient example is the current period of rapid growth in mainland China, which has been spurred by continuing movement toward a market economy.

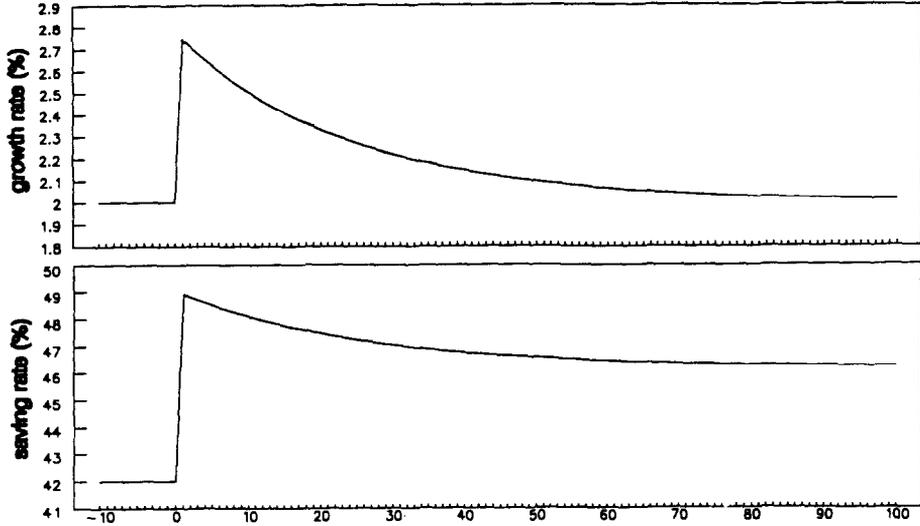
The experiment we consider is a change in λ for a country that is initially in steady state. We consider a country with an initial technology growth rate of .02, in which the growth rate is raised to .03.⁷ The results are presented

⁶In practice this experiment is identical to those considered by King and Rebelo (1993), in which countries start off with capital below their steady-state levels.

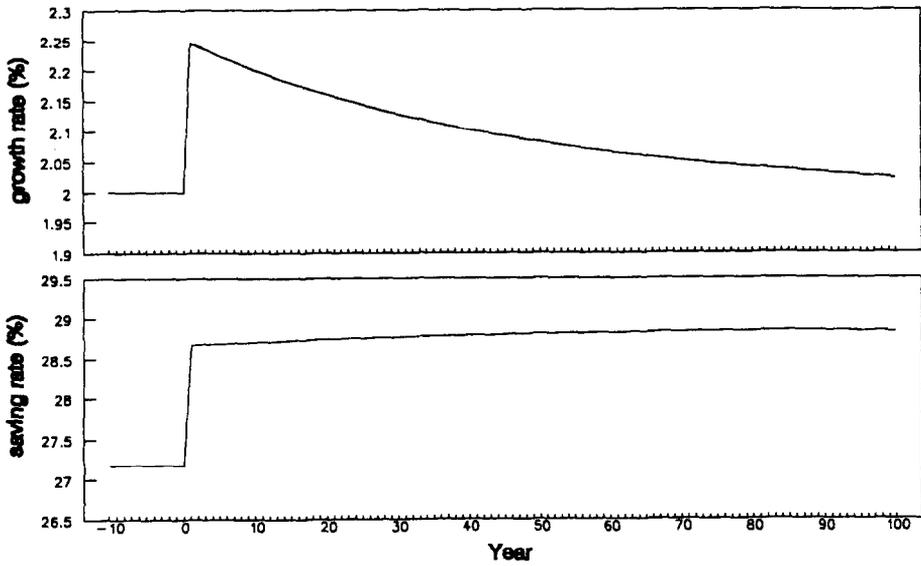
⁷Although the experiment that we consider here is a *permanent* change in the growth

Figure 1
 Response of Growth and Saving to a Shock to θ

$\rho=1$



$\rho=4$



in Figure 2. The change in the growth rate of technology produces a rising path in the growth rate of output. Output does not initially grow at the new rate of technology growth because the stock of capital per efficiency unit is initially higher than in the steady state. In the case where $\rho = 1$, the transition to the new growth rate is fairly rapid: the growth rate has risen to 2.5% within 11 years of the shock, and to 2.75% within 26 years. In the case where $\rho = 4$, the transition is slower: growth rises to 2.5% only after 31 years, and to 2.75% after 60 years.

The behavior of the saving rate here is qualitatively different from that in the previous experiment. In the case where $\rho = 1$, saving drops immediately, then begins rising, but remains lower than its initial level for eight years. Thus, at least in the medium run, the relationship between saving and growth is negative. Also, the movement of the saving rate is fairly small in comparison with the movement of the growth rate: between the steady state where growth is 2 percent and the steady state where growth is 3 percent, the saving rate only rises from 42.0% to 44.0%. In the case where $\rho = 4$, the saving rate falls in response to an increase in the growth rate and remains roughly constant below its initial level. In this case the consumption-smoothing effect dominates the substitution effect of higher interest rates. Thus, for both values of ρ the medium-run relationship between growth and saving is negative, and if $\rho = 4$ the long-run relationship is negative as well.⁸

The simulations presented above are similar to King and Rebelo's (1993) analysis of the neoclassical model. In their model, as well as in Christiano (1989), countries which start out with capital stocks well below their steady-state levels experience both rapid growth and high saving. This high saving is in turn a product of high interest rates, which compensate for the depressing influence on saving of the human-wealth effect in the presence of rapidly growing income. For example, in King and Rebelo's simulations, annual real interest rates can be higher than 50%. But in fact, interest rates to savers in many rapidly growing countries have been surprisingly *low*.⁹ Thus, we believe that the high saving rates of rapidly growing countries such as Japan remain unexplained by the neoclassical model.

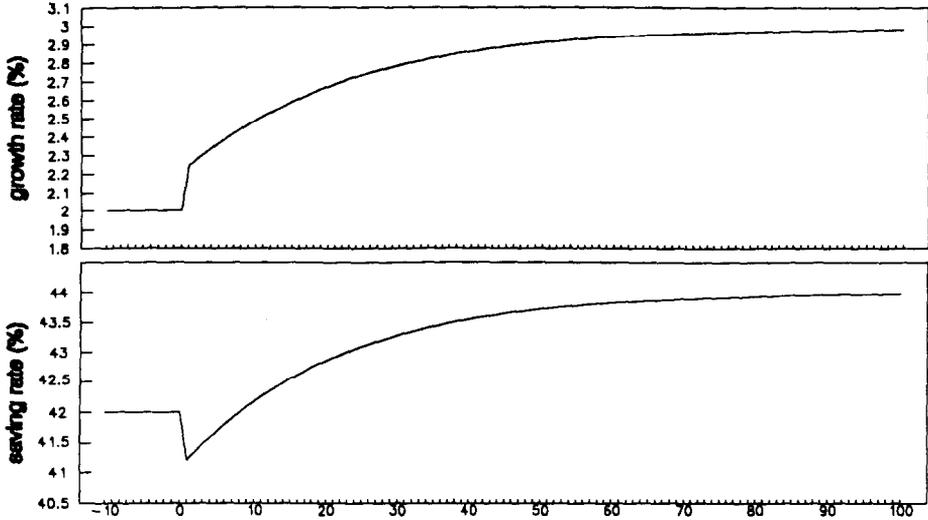
rate of technology, we also could have considered a temporary change in λ . For changes in λ lasting for a sufficiently long time, the results would be similar.

⁸Viard (1993), working analytically with a linearized version of the neoclassical growth model, reaches a similar conclusion about the effects of changes in growth rates on saving. He argues that the failure of saving rates to rise in response to the post-1973 productivity slowdown, despite the fact that published forecasts of future long-term growth declined dramatically, is strong evidence against the permanent income hypothesis.

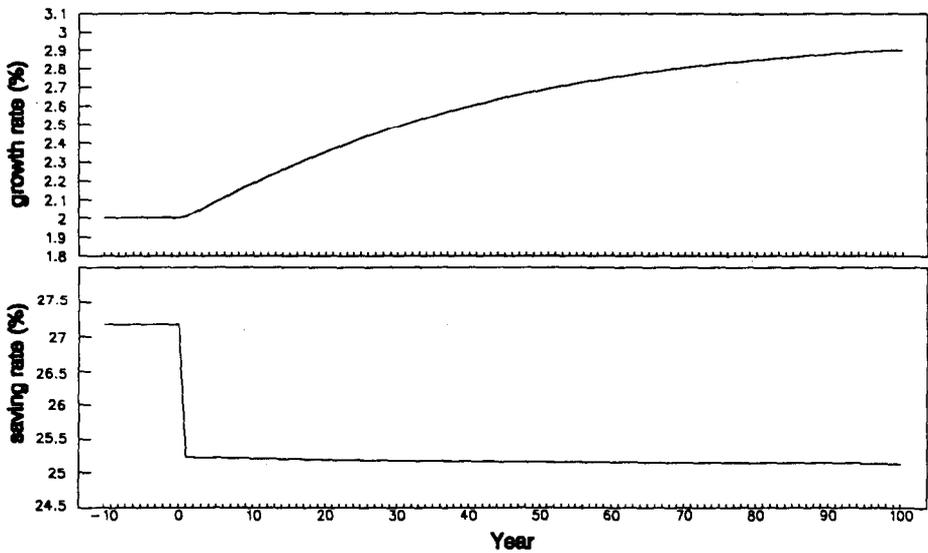
⁹Christiano reports that the return to the Japanese stock market in the 1960s and 70s was roughly as high as the return to capital in his calibrated growth model, but given that most Japanese household saving is not in the stock market, it is not clear why this return is the right measure on which to focus.

Figure 2
 Response of Growth and Saving to a Shock to λ

$\rho=1$



$\rho=4$



We take this exploration of the neoclassical model to have shown that, although the “mechanical link” from saving to growth is capable of generating a medium-run positive relationship between saving and growth, the “human-wealth” link from growth to saving means that exogenous increases in growth can be associated with declines in saving, at least over the medium run. Our next task is to test whether either of these theoretical links between saving and growth can be found in the data.

2.3 *Granger causality results*

In the theoretical model presented above, when there is a change in one of the parameters, both growth and saving change immediately. In applying the model to the data we might expect to see a less simultaneous movement. If, for example, it takes some time to adjust consumption to its new optimal level following a shock to growth, or if it takes time for consumers to understand that a shock has occurred, then when λ changes, we might expect to see the growth rate of output change first, followed by a change in the saving rate. Similarly, given that investment takes time to become productive, we might expect to see a change in the discount rate reflected first in a change in saving, and only later in a change in growth.

It is in this spirit that we look more closely at the timing of movements of saving and growth within countries. The first experiment above suggested that if there are shocks to the discount rate, then we would expect saving to Granger cause growth, with a positive sign. If there are shocks to λ , the growth rate of technology, then at least in the medium run we would expect growth to Granger cause saving, with a negative sign – the second experiment.

The data that we examine are the panel of nonoverlapping five-year averages of saving and growth examined in Table 2. Table 3 reports the results of our basic Granger causality tests. All regressions include a full set of country dummies – thus we are taking out the effect of cross-country differences in average rates of growth and saving. In addition, in some regressions we included a set of time-period dummies, and report the p -value for the test that the set of dummies is equal to zero.

In the top panel of the table, we present regressions of saving on lagged saving and lagged growth. In the OECD sample, lagged growth enters positively and significantly when year effects are excluded from the regression. When year effects are included, the coefficient on lagged growth falls only slightly but becomes insignificant, while the year effects are jointly insignificant. In the broad sample of countries, lagged growth is always positive and significant.¹⁰

¹⁰Deaton and Paxson (1992) find similar results examining time series data from Taiwan.

Table 3:
Granger Causality Tests in Levels

	Dependent Variable: s_t			
	OECD	OECD	Full	Full
s_{t-1}	.473 (.074)	.457 (.103)	.482 (.046)	.474 (.051)
g_{t-1}	.257 (.109)	.195 (.149)	.318 (.080)	.268 (.084)
time effects:		0.354		0.009
Observations	132	132	353	353
\bar{R}^2	.869	.869	.810	.817
Root MSE	.0186	.0186	.0308	.0303

	Dependent Variable: g_t			
	OECD	OECD	Full	Full
g_{t-1}	.232 (.090)	-.060 (.105)	-.059 (.068)	-.239 (.065)
s_{t-1}	-.259 (.061)	-.082 (.073)	-.117 (.039)	-.001 (.039)
time effects:		.000		.000
Observations	132	132	353	353
\bar{R}^2	.248	.459	.149	.315
Root MSE	.0154	.0131	.0260	.0233

Note: Standard errors in parentheses. All regressions include a full set of country dummies. The number in the row marked “time effects” is the p -value from the test that the coefficients on the set of included time dummies are zero. s_t and g_t are average saving and growth rates over five-year periods.

The bottom panel of Table 3 tests whether saving Granger causes growth. When year effects are excluded, saving enters negatively and significantly in both samples. When year effects are included, however, the coefficient on saving is reduced and becomes insignificant.

A potential problem with the regressions presented in Table 3 is that fixed effects regressions with lagged endogenous variables on the right-hand side will produce biased estimates in short panels. The bias results from a correlation between the residual for a given observation and the country fixed effect (See Hsiao, 1986). In this case, a consistent estimator can be obtained by running the regression in differenced form and using the twice-lagged difference of the dependent variable to instrument for the once-lagged difference. We present such estimates for our Granger causality tests in Table 4. The only statistically significant result in this table is that, for the full sample of countries, changes in the growth rate of output Granger-cause changes in the saving rate with a positive sign.

The most surprising result of these exercises is that growth Granger causes saving with a positive sign. This finding is consistent with our cross-country findings in Tables 1 and 2, but not with the consumption model underlying the neoclassical growth model. The second experiment above showed that, if changes in growth rates are expected to persist, changes in growth should have a negative effect on saving, at least over the medium-run time frame considered here.

The second empirical result — that if there is any causality running from saving to growth, it is with a negative sign — is also interesting. This result is inconsistent with the common view that the reason cross-country regressions show a positive association between saving and growth is that high saving *produces* high growth via the mechanical link from saving to capital and capital to output. On the other hand, this result may not be inconsistent with the optimal growth model if consumers have advance knowledge about income growth rates. The logic is that of Campbell (1987), who argues that consumption should go down in advance of a decline in income if the income drop was anticipated (this is just the human-wealth effect on consumption). Examining quarterly U.S. data, Campbell confirms the prediction that saving Granger causes income growth with a negative sign. A problem with Campbell's results, however, is that they could have been produced by a Keynesian model with completely myopic consumers whose consumption function was subject to stochastic shocks. A positive shock to saving would reduce aggregate demand and therefore cut income in subsequent quarters. Over longer horizons such as our five-year periods, the aggregate demand effect of increased saving should be attenuated, but the mechanical link between saving and growth should begin to bite, leaving the net prediction of the model ambiguous.

Table 4:
Granger Causality Tests in Differences

	Dependent Variable: $s_t - s_{t-1}$			
	OECD	OECD	Full	Full
$s_{t-1} - s_{t-2}$.207 (.544)	-.567 (1.34)	-1.08 (.528)	-1.03 (.499)
$g_{t-1} - g_{t-2}$.042 (.257)	.139 (.428)	.459 (.184)	.352 (.167)
time effects:		0.041		0.027
Number of Obs.	88	88	228	228
Root MSE	.02266	.0237	.05597	.05332
	Dependent Variable: $g_t - g_{t-1}$			
	OECD	OECD	Full	Full
$g_{t-1} - g_{t-2}$	-.236 (.230)	-.610 (.174)	-.071 (.128)	-.073 (.123)
$s_{t-1} - s_{t-2}$	-.175 (.136)	.002 (.103)	-.058 (.062)	-.062 (.062)
time effects:		0.000		0.049
Number of Obs.	88	88	225	225
Root MSE	.01798	.01428	.03259	.03218

Note: Standard errors in parentheses. In each regression, the lagged difference in the dependent variable is instrumented with the twice-lagged difference of the same variable. All regressions include a constant, but not country dummies. The number in the row marked "time effects" is the p -value from the test that coefficients on the set of included time dummies are zero. s_t and g_t are average saving and growth rates over five-year periods.

Our conclusion is that neither of the simple causal linkages between growth and saving explored in our theoretical experiments explains our data,¹¹ but there is nevertheless an important empirical linkage: higher growth leads to higher saving.

2.4 *Single country case studies*

As another way of looking at the relationship between saving and growth in aggregate data, we examine data from individual countries. We focus on a set of high-saving, high-growth East Asian countries whose experiences have been crucial in shaping the growth literature. For example, excluding Japan from the saving regression for the OECD sample in the second column of Table 1 reduces the coefficient on growth from 4.73 (standard error of 1.23) to 3.34 (1.55). Similarly, excluding Japan, Hong Kong, and South Korea from the regression for the full sample in the fourth column reduces the coefficient on growth from 1.06 (.54) to .62 (.69).

In Figures 3 through 6 we look directly at data from Japan, South Korea, Singapore, and Hong Kong to see what can be learned about the relation between saving and growth.¹² For each country we plot time series of centered three-year averages of the growth rate and the saving rate. The message of these four figures is, to us, fairly unambiguous: in all four cases, growth was high early, and saving was high later. In South Korea, for example, over the period 1960–74, growth averaged 6.1% while average saving was only 10.4%. Over the period 1975–87, growth averaged 5.3%, while saving averaged 27.8%. In none of the countries does it appear that large increases in the saving rate were reflected in subsequent high growth.

The data from these countries are consistent with our Granger causality results that high growth is followed by, rather than preceded by, high saving. Since these countries are to such a large extent the determinants of the cross-country result that growth and saving are highly correlated, this examination of time-series data casts further doubt on the conventionally accepted wisdom that the growth-saving correlation is driven by causality running from saving to growth.¹³

¹¹To avoid misinterpretation, we should emphasize that, despite the results of Tables 3 and 4, we both still believe that an exogenous increase in the saving rate would lead to an increase in economic growth. The argument here is only that the observed pattern of data could not have been generated by a neoclassical model in which the primary shocks were exogenous changes in the saving rate.

¹²Data is from Summers and Heston (1991). See the Data Appendix for the definition of saving. Singapore is not included in the regressions in Table 1 because it does not have data for all of the years 1960–87.

¹³In some of the countries we examine, high growth seems to produce not only a high saving rate, but a *constantly rising* saving rate. Although it is probably true that no simple model can explain all that is going on in these countries, the partial equilibrium-

Figure 3
Saving and Growth in Japan

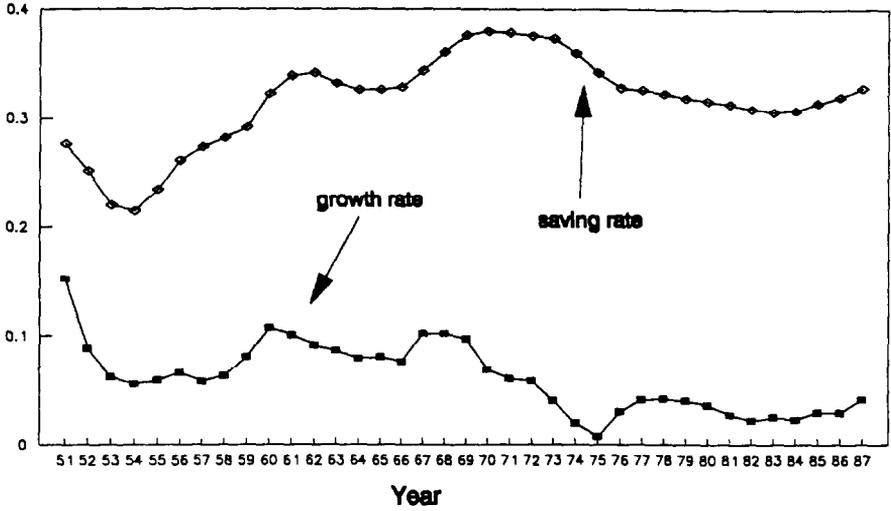


Figure 4
Saving and Growth in South Korea

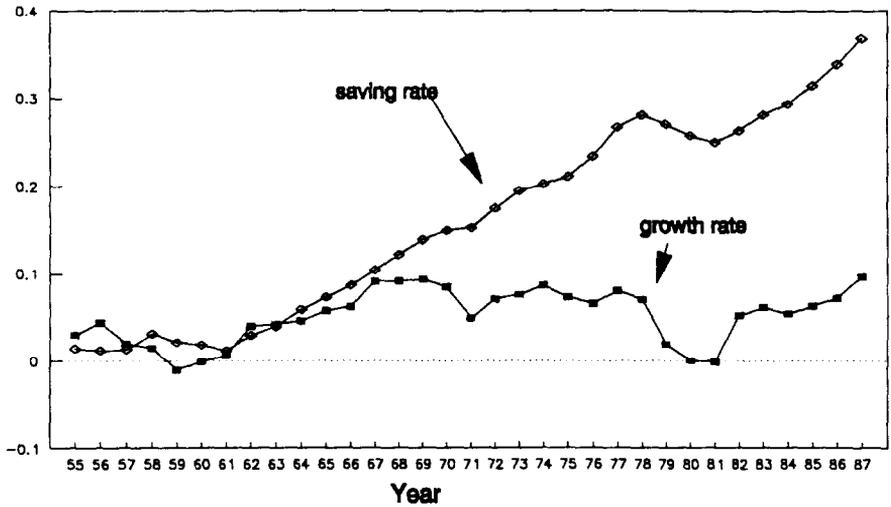


Figure 5
Saving and Growth in Singapore

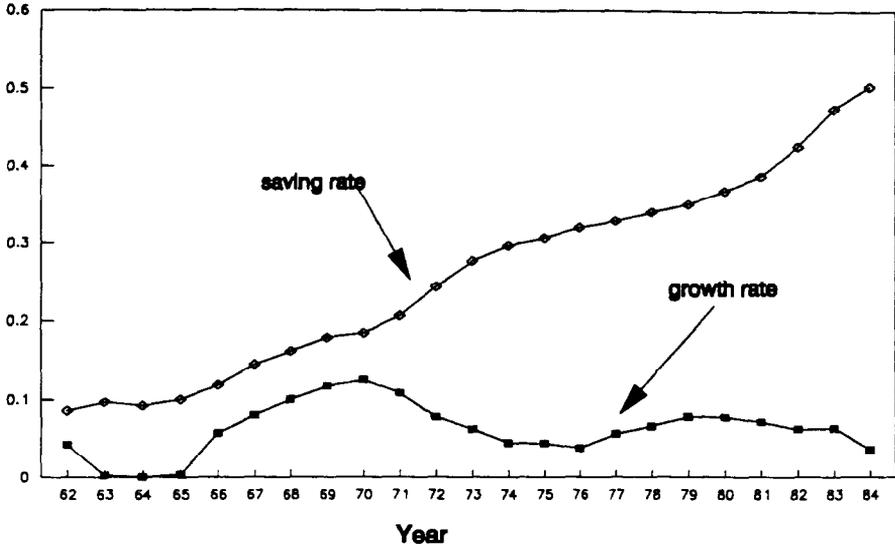
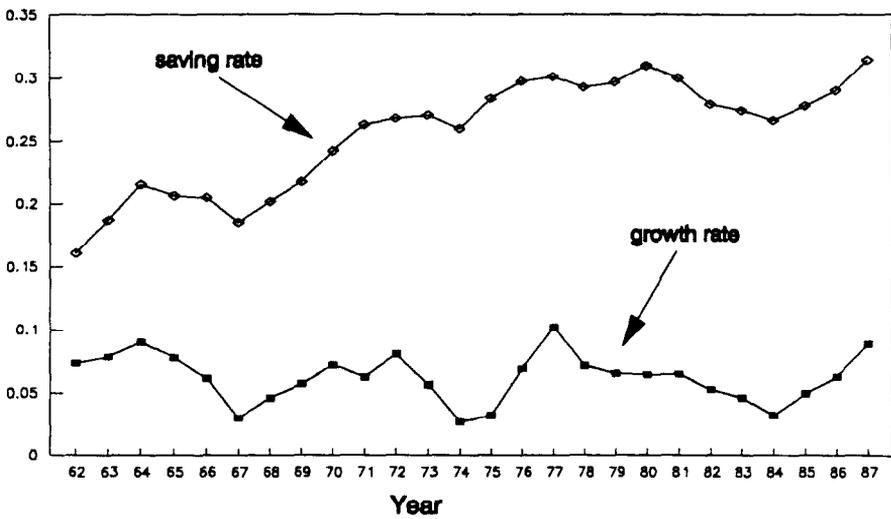


Figure 6
Saving and Growth in Hong Kong



2.5 *Conclusions from the aggregate evidence*

The recent literature on economic growth has typically explained the positive cross-country correlation between saving and growth as the result of high saving producing high growth via capital accumulation. Our empirical results suggest, however, that higher growth *precedes* higher saving. Furthermore, higher saving is not followed by higher growth, at least in the medium run. If our evidence is convincing, it has implications for both the theory of consumption and for the analysis of economic growth. We address these implications in Sections 4 and 5, respectively. Before doing so, however, we examine the relation between saving and growth in household-level data, where the general equilibrium effects that bedevil analysis of aggregate-level data are not present.

3 **The household evidence**

In this section we turn to household-level data to examine the relationship between income growth and saving. The question we hope to answer by looking at household data is whether people who have predictably high income growth save more or less than people who have predictably low income growth. To address our question we use data from three household surveys, the *Panel Study of Income Dynamics* (PSID), the 1983 *Survey of Consumer Finances* (SCF), and the 1961–62 *Consumer Expenditure Survey* (CEX). The basic technique will be to construct estimates of predicted income growth for each household based on the age, occupation, and education of the household head. We then construct estimates of the saving rate or the wealth/income ratio for each family, and then regress this measure on predicted income growth. In all three data sets we find a highly statistically significant positive relationship between saving or wealth and predictable income growth. We also find that the level of saving or wealth is positively related to the level of permanent income, but even controlling for the effect of permanent income on saving we generally find that households with predictably greater income growth save more.

3.1 *The PSID evidence*

Our extract from the PSID contains data on income for a sample of households from 1968 through 1987. Although there is no direct measure of the saving rate in the survey, in 1984 households were surveyed about their wealth holdings. Abstracting from capital gains and losses, wealth must come either from saving by the household itself or from transfers of wealth from other

habit-formation model that we present below can produce such a phenomenon in some cases. See in particular Figure 11b.

households. We restrict our sample to households which have never received an inheritance, and *inter vivos* transfers are included in our definition of household income, so observed wealth in 1984 for our sample of households should correspond at least roughly to past saving out of total income. We further restricted our sample to households with heads between the ages of 30 and 40 in order to examine households at an early stage of the life cycle when the predictions of consumption models are clearest. (See Section 4 for an overview of those theoretical predictions.)

We base our crude saving measure for each household on the ratio of wealth at the end of 1983 to average income over the 1981–87 period for the household. Since wealth/income ratios appear to be approximately log-normally distributed over most of their range, we wanted to take the log. However, net worth is zero or negative for about 5–10 percent of the sample, precluding a logarithmic specification. Our solution was to add one to the W/Y ratio before taking the log. Thus, our dependent variable is $S = \log(W/Y + 1)$.

We observe income in each year for each household. To avoid conceptual problems associated with changes in marital status, we restricted the sample to households whose marital status did not change over the period. Because the income of farmers and the self-employed is much more variable, and more difficult to measure correctly, than that of people in other occupations, we excluded the self-employed and farmers from all our results. After dropping observations with missing wealth, education, or occupation information, and making a few other sample restrictions (see the Data Appendix for details), we were left with a total of 287 observations.

Before turning to the econometric estimates, we present some simple plots of the data. Figure 7a plots the average values of the growth rate of income and of our S variable by education group for our PSID sample. We could have simply plotted the six (growth, wealth) combinations for the six education groups, but if we had done so it would not have been possible by looking at the graph to tell how many households were in each of the six education groups. Therefore, for each education group we plotted a cloud of points randomly distributed around the group mean, where the number of points in the cloud was equal to the number of households in the group. The figure shows a strong positive association between income growth and the wealth/income ratio across people in different education groups. The next figure performs the same experiment using the six occupational categories we consider, and also shows a strong positive relationship between growth and saving.

We turn now to more formal econometric tests. The prototype equation we wish to estimate in all three datasets is:

$$S = \delta_0 + \delta_1 g + \delta_2' Q + e \tag{4}$$

Figure 7a
Growth vs. $\text{Log}(W/Y + 1)$, By Education (PSID)

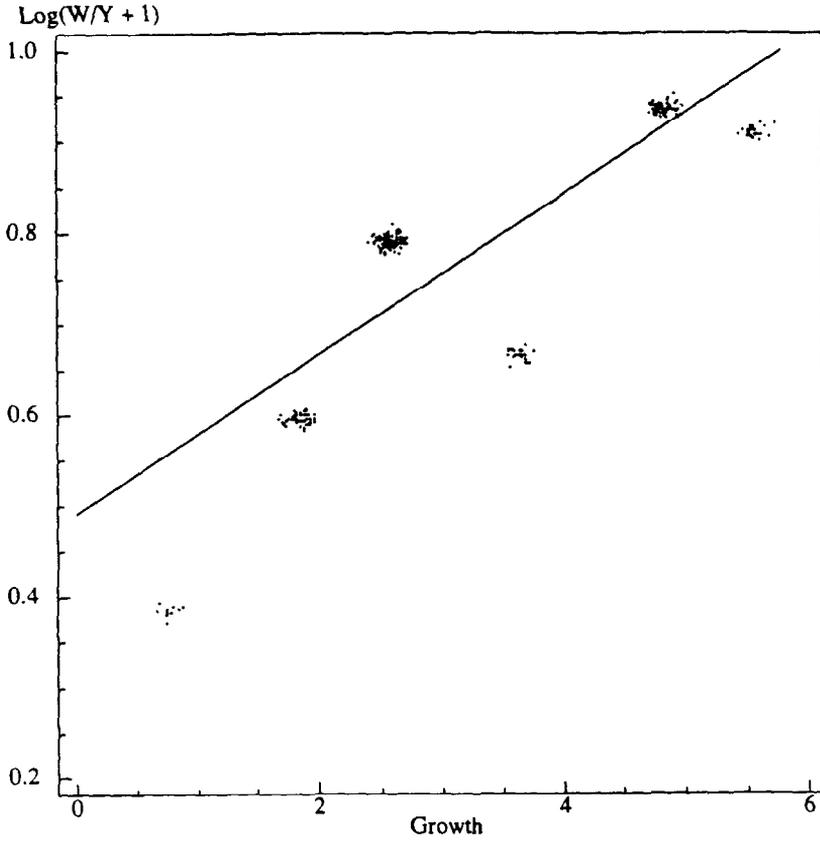
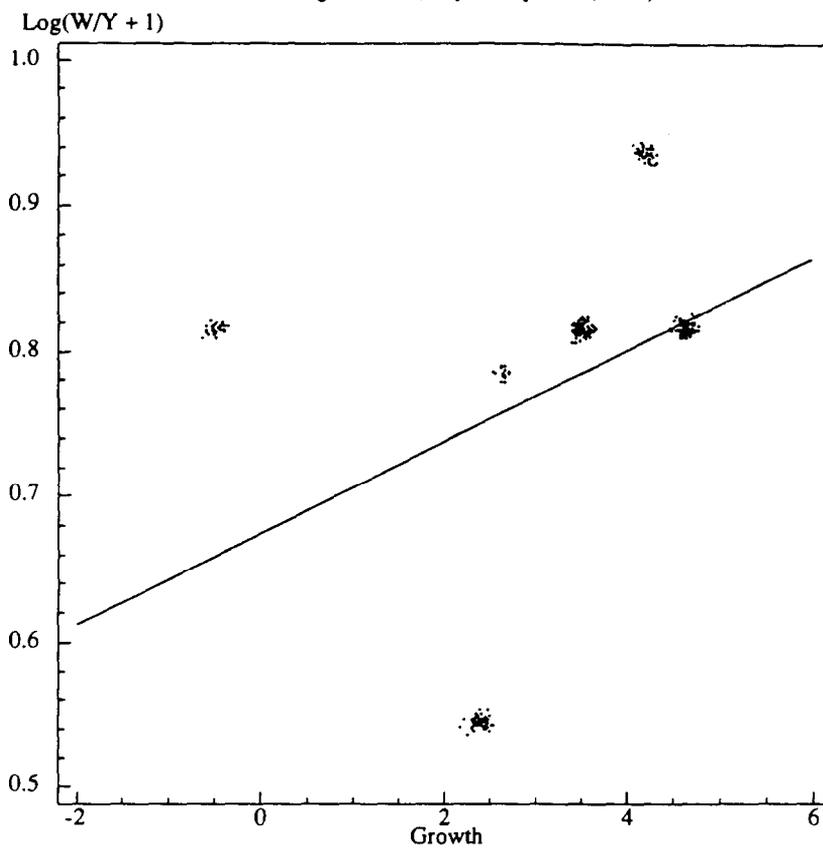


Figure 7b
Growth vs. Log(W/Y + 1), By Occupation (PSID)



or, combining a constant, g , and Q into a matrix X :

$$S = X\delta + e \tag{5}$$

where S is the measure of saving, g is the predictable component of income growth, and Q is a set of other variables that might plausibly be related to the saving rate. Specializing this equation to the PSID case, S will be $\log(W/Y + 1)$ and g will be the predicted growth rate of income. The key coefficient, δ_1 , shows the effect of income growth on saving.

We do not observe households' predictions for income growth directly. What we do observe, the actual growth rate of income for each household over the 1981–87 period, is presumably the sum of the predictable component of income growth and an error term. If we were to perform regression (4) using actual income growth, therefore, we would expect the coefficient on δ_1 to be biased. If the prediction error were uncorrelated with saving, this would be a classic errors-in-variables problem, and δ_1 would be biased downward. If, however, the prediction error represented transitory shocks to income, the LC/PIH model would imply that almost all of the shock should be saved, i.e., the error would be positively correlated with S . This amounts to a simultaneity problem.

The solution to both errors-in-variables and simultaneity problems is to estimate the equation using instrumental variables. The instruments used are the same education and occupation variables used for the plots above, along with the age of the household head. In our basic specification, the only control variable in Q is the age of the household head.

The results are presented in Table 5. Regression (1) finds that the coefficient on income growth is 4.69 with a heteroskedasticity-robust standard error of 1.57, which is significant at better than the 1-percent level. This coefficient implies that a one-percentage point increase in the predictable growth rate of income would produce an almost 5-percentage-point increase in the wealth/income ratio.

Using education and occupation as instruments in regression (1) implicitly assumes that the only channel through which occupation and education affect wealth is through their effect on the growth rate of income. One might suspect that education and occupation are correlated with saving through other channels as well. For example, if people with higher permanent incomes save more, *ceteris paribus*, and if education and occupation are correlated with the level of permanent income, then the identification assumptions of the model in regression (1) are wrong and the coefficient on the income growth term could be biased.

Hansen's (1982) test of overidentifying restrictions is designed to detect precisely this kind of problem. We therefore present the p -value for the test of overidentifying restrictions for our model in the second-to-last column of

Table 5

Regressions of $\log(W/Y+1)$ on Expected Growth in the PSID and the SCF
(Standard Errors in Parentheses)

Regression	Data Set	Intercept	Expected Growth	Age of Head	Log(Income)	p-value for test of Homoskedasticity	p-value for QJD test	Number of Observations
(1)	PSID	-0.22 (0.34)	4.69 *** (1.57)	.025 (0.010)		0.68	0.08	287
(2)	PSID	-4.58 (1.24)	2.75 * (1.64)	.012 (0.012)	0.48 (0.14)	0.01	0.35	287
(3)	SCF	-1.06 (0.47)	6.76 *** (1.94)	.051 (0.014)		0.18	0.05	463
(4)	SCF	-5.88 (1.39)	4.31 ** (1.91)	.037 (0.013)	0.53 (0.14)	0.58	0.89	463
(5)	Pooled PSID and SCF	-0.77 (0.33)	6.08 *** (1.43)	.042 (0.010)		0.14	0.31	750
(6)	Pooled PSID and SCF	-5.45 (0.99)	3.80 *** (1.37)	.028 (0.009)	0.51 (0.10)	0.46	0.20	750

Notes:

Sample selection and data are described in the Data Appendix.

(1) and (2) are estimated by 2SLS with the White heteroskedasticity correction.

(3)-(6) are estimated using the TS2SLS methodology described in the Technical Appendix.

The QJD test is derived from Hansen's (1982) test; see the Technical Appendix.

* Significant at the 10 percent level.

** Significant at the 5 percent level.

*** Significant at the 1 percent level.

Table 5. The p -value of .08 rejects the specification at the 10-percent level, suggesting that our instruments do indeed have some explanatory power for wealth independent of their ability to predict income growth. We therefore added to our specification the natural control variable suggested above: income. Regression (2) of Table 5 presents the results when the log of average 1981-87 income is added to regression (1). This specification appears to fit the data substantially better than the specification of regression (1): the OID test now passes at a significance level of .35, and the coefficient on income growth is reduced but still significant at the 10-percent level.

3.2 *The SCF and the pooled PSID/SCF evidence*

The PSID was not designed to collect data on wealth, and although some studies have found that the wealth data in the PSID are reasonably good (see, for example, Curtin, Juster, and Morgan (1989)), it would lend credence to our PSID results if we found similar evidence in a survey explicitly designed to measure wealth. We therefore turned to the 1983 *Survey of Consumer Finances* of the Federal Reserve Board to conduct further tests.

The SCF is deficient relative to the PSID in one respect, however: it contains data on only a single year of income. It would therefore be difficult to construct an estimate of expected income growth using only data from within the SCF (although not impossible; see below for the discussion of our income growth estimates in the CEX). Our solution was to estimate the relationship between income growth and the instruments (education, occupation, and age) in the PSID, and then to use the PSID income growth equation to predict income growth for the SCF consumers. This amounts to running the first-stage regression of a Two-Stage Least Squares estimation in the PSID and the second-stage regression in the SCF, a procedure we call Two-Sample Two-Stage Least Squares (TS2SLS). (This is a specialization of the Two-Sample Instrumental Variables (TSIV) technique described in Angrist and Krueger, 1990. See the technical appendix for details of the estimation procedure.)

To be concrete, call our instrument set Z , where Z contains the education, occupation, and age variables described above. The goal is to estimate δ in the equation $S = X\delta + e$ even though we do not observe X (or at any rate all elements of X) in the SCF. However, we observe the values of instruments Z , and we can estimate the following (first-stage) regression in the PSID:

$$X = Z\alpha + \nu \tag{6}$$

Estimating α in this equation yields:

$$a = (Z'Z)^{-1}(Z'X) \tag{7}$$

In the SCF we can then construct $\hat{X} = Za$, and then we can estimate the equation:

$$S = \hat{X}\delta + u \tag{8}$$

by OLS in the PSID, yielding a consistent estimate for δ . Obtaining a consistent estimator for the asymptotic variance-covariance matrix of δ is somewhat more difficult, and that exercise is relegated to the technical appendix.¹⁴

Before presenting our results we should mention some minor differences between the variables and the samples in the SCF and the PSID. In the SCF the dependent variable S is again defined as $\log(W/Y + 1)$, but W is net worth at the end of 1982 (rather than 1983 as in the PSID), and Y is noncapital income in 1982 (rather than the average of noncapital income over the 1981-87 period). As in the PSID, the SCF sample consists of married households whose head is between the ages of 30 and 40. In addition, to construct the W/Y variable we had to restrict the sample to households with positive labor income in 1982. For further sample restrictions, see the Data Appendix.

The results from estimating δ using TS2SLS are presented in regressions (3) and (4) of Table 5. In general the results are similar to those from the PSID, but the coefficient on income growth is somewhat larger and more statistically significant, comfortably exceeding the 5-percent significance threshold in both cases. The greater statistical significance probably results, in part, from the larger sample size of the SCF dataset. As in the PSID, the OID test reveals evidence that the version of the equation which does not include income as an independent variable is misspecified, but once income is included (in regression 4) the OID test provides no further evidence of misspecification.

Once the model has been estimated separately in both the SCF and the PSID, it is a simple matter to estimate it using the pooled data from both datasets. All that is required is to stack the data on S and \hat{X} from both datasets and estimate the stacked system via OLS. Deriving consistent standard errors is only slightly more difficult (again, see the technical appendix). The results of such a pooled estimation are presented in regressions (5) and (6) of Table 5. As one would expect, the coefficient estimates fall between those of the PSID alone and those from the SCF alone, and the statistical significance of the coefficients is greater than that in either dataset alone.

¹⁴We should note here, however, that (contrary to an apparent claim in Angrist and Krueger (1990)), in order to construct a consistent variance-covariance matrix for δ in the simultaneous-equations case where X is correlated with e , it is necessary that at least one of the two datasets contain observations on all three of S , X , and Z . Since the PSID contains all three of S , X , and Z , we are able to construct a consistent estimate of the variance-covariance matrix.

3.3 Evidence from the 1961–62 Consumer Expenditure Survey

The evidence from both the PSID and the SCF relies on estimated income growth from a single data source, the PSID, over a single time period, 1981–87. If this was an atypical period for the relationship between education, occupation, and income growth, our results in Table 5 could be spurious. This is of particular concern because extensive research in the labor economics literature has found that the returns to education and other observable characteristics were changing over this time period (see, for example, Katz and Murphy (1992)). Another problem with the PSID and SCF analysis is that neither data set contained a direct measure of the saving rate. To further check the robustness of the relationship between income growth and saving across households, we decided to look at data from another data source covering a very different period: the 1961–62 *Consumer Expenditure Survey*.

In addition to covering a different time period, the CEX has the virtue of containing a direct measure of the saving rate for each household in the survey. Households were asked their income, their consumption, and their saving for the survey year and (unlike in subsequent consumer expenditure surveys), a real effort was made to educate households about the balance-sheet relationship between consumption, income, and saving. As a result, discrepancies between the quantity (income - consumption) and reported saving are much smaller than in subsequent consumer surveys.

The chief disadvantage of the CEX is that it does not contain any panel data on income growth. We therefore construct an estimate of predicted income growth for each household by looking at the income of households in similar occupational and educational categories who are farther along in the life cycle. Carroll (1994) also used this technique and found that income growth forecasts constructed in this manner using the PSID performed relatively well in comparison with actual subsequent income experience.

The model of income is as follows:

$$y_i = \sum_j D_{i,j}[\pi_j + Age_i\gamma_j] + u_i \quad (9)$$

where y_i is the log of real labor income, $D_{i,j}$ is a set of dummy variables indicating household i 's occupation and educational group, and Age_i is the household head's age. This framework allows the estimation of a different intercept π_j and growth rate γ_j for each dummy variable j . Grouping all the dummy variables for household i into a single row vector D_i , and grouping the coefficients γ into a column vector, we have:

$$y_i = D_i\pi + Age_iD_i\gamma + u_i \quad (10)$$

which can be estimated by OLS. The projected income growth rate for household i is thus given by $\hat{g}_i = D_i\hat{\gamma}$. These projected growth rates can then be

used to estimate the CEX version of equation(4). This procedure bears some resemblance to two-stage least squares estimation, but differs because the value of the income growth term g is never directly observed, even in the pseudo-first-stage regression of equation (10).

The sample restrictions for estimating equation (10) were similar to those used for the PSID and SCF. (For exact sample restrictions, see the Data Appendix). In addition we had to decide the appropriate restrictions to place on the age of the head of household. In the end we estimated the equation two ways, first restricting the sample to households whose head was between the ages of 30 and 40 (as in the PSID and SCF), and second restricting the sample to households whose head was between the ages of 30 and 60. The expected result of the former technique should be to produce the projected growth rate of income only during the early stage of the life-cycle (henceforth designated \hat{g}_{young}), while the second technique should produce an estimate of the growth rate of income over essentially the household's working lifetime (\hat{g}_{life}).

As with the PSID, we present a simple plot of the data before we turn to formal estimation. (The same technique as in Figure 7 was used to generate randomized predicted growth and saving by group.) Figures 8a and b plot the average value of \hat{g}_{young} against average saving rates for young households (with heads aged 30 to 40) in the six education groups and the six occupation groups in our sample. Figures 9a and b plot \hat{g}_{life} versus average saving for the same households. In all the figures there is a positive association between the projected growth rate of income and the saving rate. However, saving appears in these figures (and in the more formal econometric results below) to be more closely related to projected lifetime growth than to projected current growth.

Our explanation of this puzzle is that \hat{g}_{life} is simply a better measure of the predictable part of income growth (even for young consumers) than \hat{g}_{young} . Intuitively, the quality of our estimates of $\hat{g}_{young} = D\gamma_{young}$ and $\hat{g}_{life} = D\gamma_{life}$ will depend on the accuracy with which γ_{young} and γ_{life} are estimated. This can be gauged by estimating equation (10) constraining γ to be zero and comparing the results to those for the unconstrained estimation described above. We performed such a regression and found that allowing different growth rates of income by occupation and education group (i.e., allowing a nonzero γ_{young}) only raises the R^2 of the regression from .229 to .235; this increase is not even close to statistically significant, having a p -value of .36. Of course, this result does not imply that there were no differences in income growth by education or occupation group for young consumers in 1961–62, but it does indicate that the methodology described in equation (10) was not powerful enough to reliably identify whatever cross-group differences did exist.

Figure 8a
Current Growth vs. Saving, By Education (CEX)

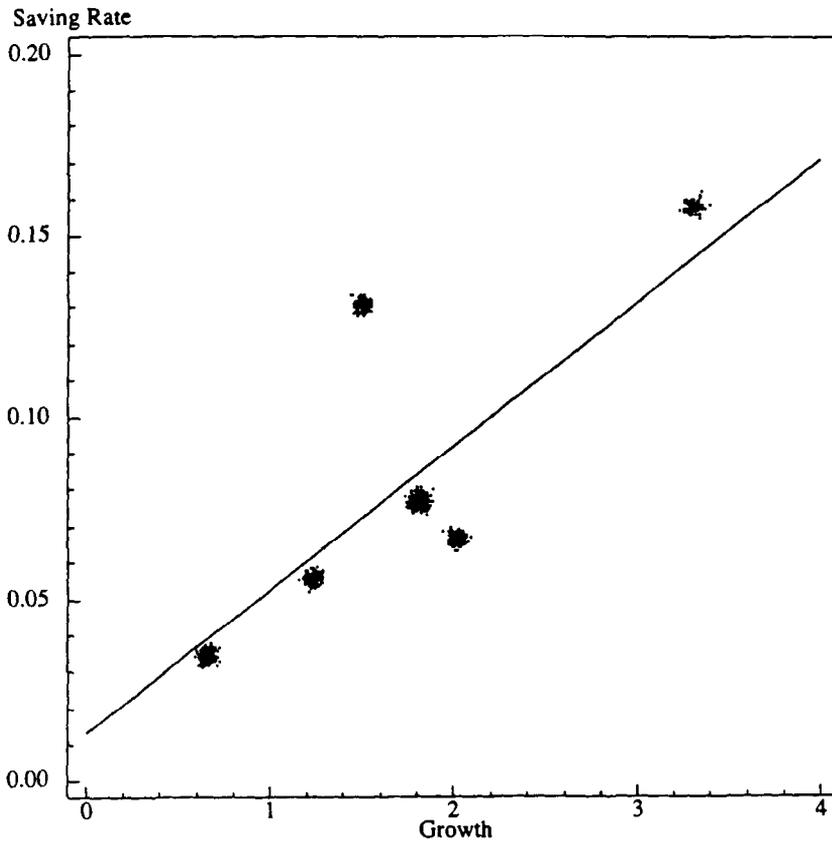


Figure 8b
Current Growth vs. Saving, By Occupation (CEX)

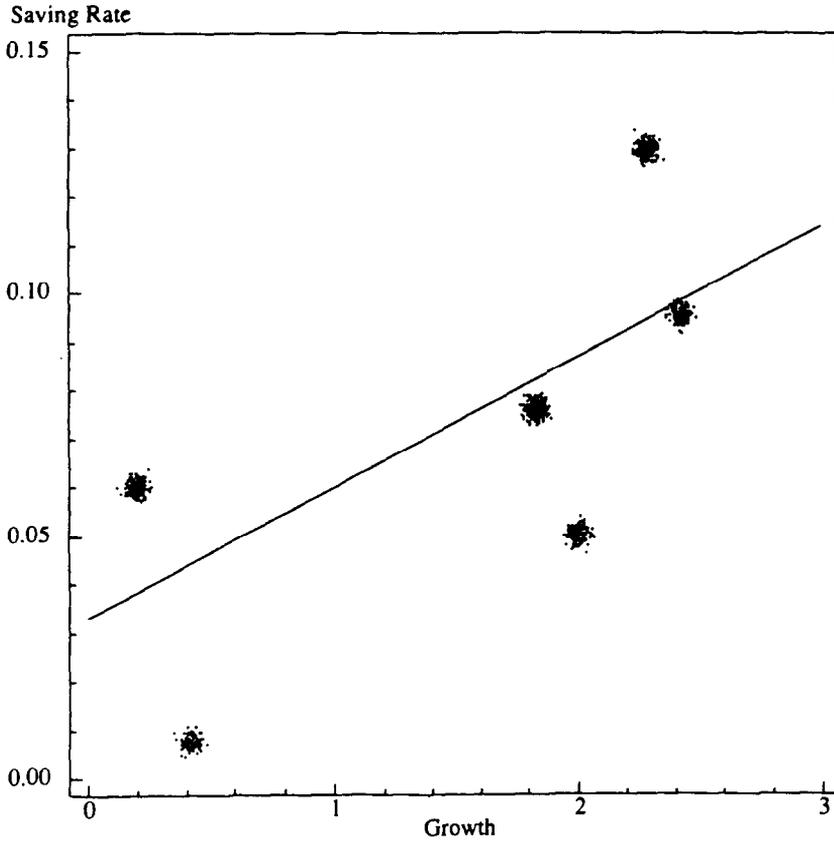


Figure 9a
Lifetime Growth vs. Saving, By Education (CEX)

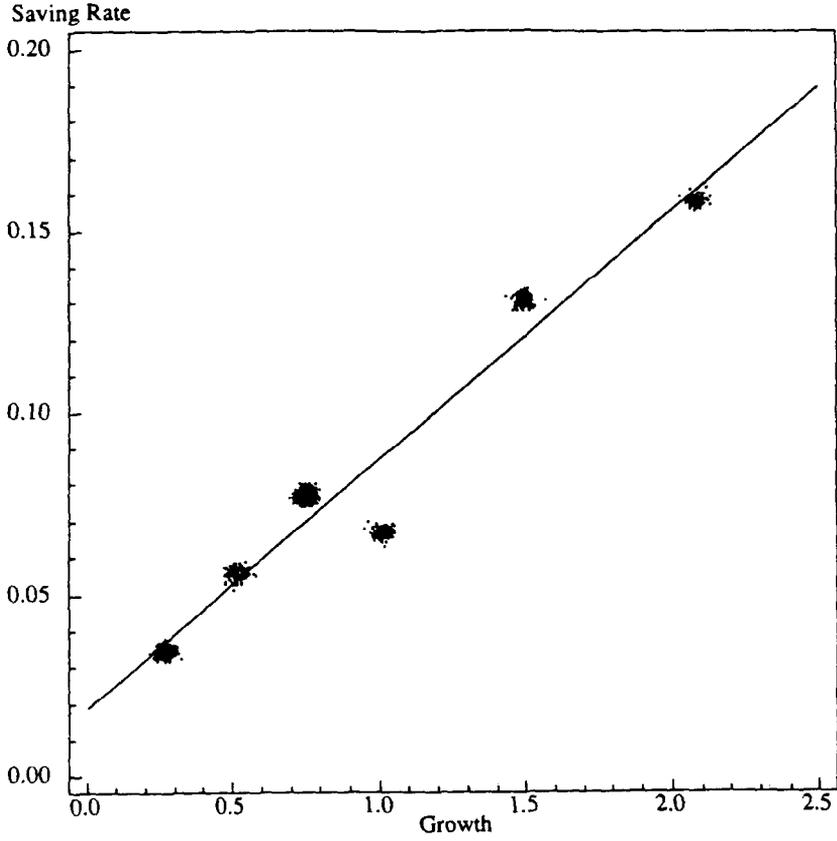
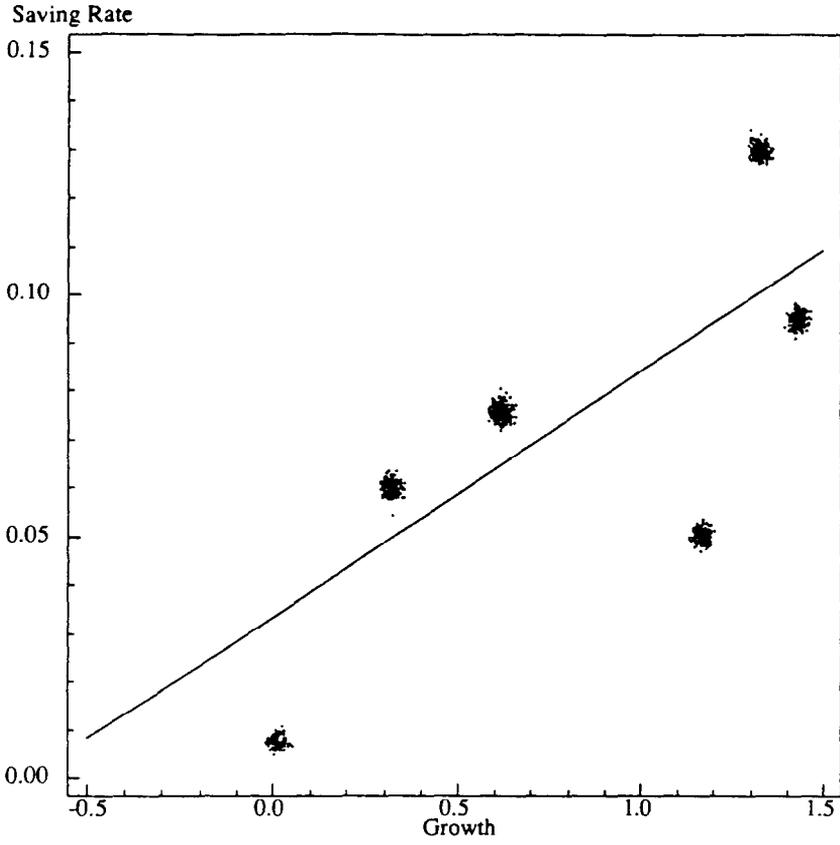


Figure 9b
Lifetime Growth vs. Saving, By Occupation (CEX)



The results for \hat{g}_{life} were much better. Allowing a nonzero value of γ_{life} raises the R^2 of the income prediction regression by about .01, an amount that is statistically significant at considerably better than the 1-percent level. It is our view, therefore, that \hat{g}_{young} is a poorer estimate of the true income growth rate than is \hat{g}_{life} , even for young households.

We proceed now to the estimation of equation (4) using the constructed values of \hat{g}_{young} and \hat{g}_{life} . Regressions (1) and (2) of Table 6 repeat the experiment of regressions (1), (3), and (5) of Table 5, regressing current saving on projected income growth and current age. As was true in Figures 8 and 9, saving is positively associated with both measures of income growth, but the association is substantially stronger with \hat{g}_{life} than with \hat{g}_{young} . We suspect, as implied above, that the apparent stronger association with lifetime growth is merely the result of superior measurement of \hat{g}_{life} relative to \hat{g}_{young} . Nelson and Startz (1990) have shown that for traditional instrumental variables estimation a poorly performing first-stage regression can generate poor results in the second-stage regression, and we believe that the poor performance of \hat{g}_{young} in these regressions may reflect a similar problem here.

Regressions (3) and (4) add the log level of income as an explanatory variable to the model, as in regressions (2), (4), and (6) in Table 5. The equation is now estimated using instrumental variables, where the instruments are the same age, occupation, and education variables used to estimate equation (10). In contrast with the results in Table 5, the income growth terms are not statistically significant once the level of income is controlled for.

The final two regressions substitute the log of consumption for the log of income, because under the null hypothesis that consumers behave according to the permanent-income hypothesis, consumption should be a better proxy for permanent income than is actual income (using consumption was not possible in the previous regressions because the other data sets contained no data on consumption). Although the coefficient on \hat{g}_{young} increases, it does not become significant. However, the coefficient on \hat{g}_{life} returns to near its level in regression (2), and is statistically significant at the 5-percent level.

Our conclusion from the CEX regressions is that the positive association between saving and growth we found in the PSID and SCF is not an artifact of the particular time period covered in those data, or of the particular measure of saving used. The CEX results provide less unequivocal support for the existence of a positive effect of growth on saving after the level of permanent income is controlled for, but certainly do not provide any reason to believe that the results from the other data sets were spurious. Finally, there is certainly no support in these data for the prediction of the permanent-income hypothesis that income growth should have a *negative* effect on saving.

Table 6

Regressions of Saving Rate on Expected Growth in the 1961-62 CEX
(Standard Errors in Parentheses)

<u>Regression</u>	<u>Intercept</u>	<u>Expected Current Growth</u>	<u>Expected Lifetime Growth</u>	<u>Age of Head</u>	<u>Log(Y)</u>	<u>Log(C)</u>	<u>p-value for QJD Test</u>	<u>Number of Observations</u>
(1)	-0.02 (0.08)	2.48 *** (0.68)		.0015 (.0022)			.001	1317
(2)	-0.03 (0.08)		5.37 *** (1.10)	.0017 (.0022)			.034	1317
(3)	-1.95 (0.45)	0.53 (0.81)		-.0014 (.0021)	0.23 (0.05)		.069	1317
(4)	-1.65 (0.65)		1.67 (1.86)	-.0009 (.0023)	0.19 (0.08)		.083	1317
(5)	-1.81 (0.70)	1.21 (0.95)		-.0009 (.0027)		0.21 (0.08)	.054	1317
(6)	-0.91 (0.86)		3.93 ** (1.90)	.0005 (.0026)		0.11 (0.10)	.061	1317

Notes:

Data sample and variable construction are described in the Data Appendix.
Dependent variable is $\log(\text{disposable income}) - \log(\text{consumption expenditures})$.

* Significant at the 10 percent level.

** Significant at the 5 percent level.

*** Significant at the 1 percent level.

4 Interpretation and discussion

This section will consider whether any standard modification of the life-cycle/permanent-income hypothesis model is consistent with our empirical results. We focus here primarily on the household-level results because we view them as the simpler and sharper challenge to the standard model, although we will occasionally appeal to the aggregate results to bolster our arguments.

To fix the analytical framework, we begin by showing formally that the usual rational expectations LC/PIH model without income uncertainty predicts a negative correlation between growth and saving, at least for young consumers. We then consider whether modifying the model to incorporate liquidity constraints, self-selection, precautionary saving, or habit formation can potentially explain a positive correlation. We are able to find circumstances under which some of these modified versions of the model can generate a positive correlation between growth and saving, but none of the modified models is fully satisfactory. In the end, we speculate that a combination of habit formation and income uncertainty may provide the best explanation for our results.

4.1 *The life-cycle/permanent-income hypothesis model*

We consider a standard Life Cycle/Permanent Income model:

$$\begin{aligned} \max \sum_{i=t}^T \beta^{i-t} u(C_i) \\ \text{s.t. } W_i &= RW_{i-1} + Y_i - C_i \\ Y_i &= GY_{i-1} \end{aligned} \tag{11}$$

where the gross interest rate $R = (1 + r)$, the gross income growth rate $G = (1 + g)$, and initial wealth and income W_t and Y_t are given. If there is no income uncertainty and the utility function is homothetic, this model can be solved for the optimal level of consumption at age t :

$$C_t = k_t [RW_{t-1} + H_t] \tag{12}$$

where H_t is human wealth and k_t is a function of the taste parameters of the consumer's utility function, the real interest rate, and other features of the problem. Crucially, k_t is *not* a function of G . If the consumer faces an infinite horizon, the expression for human wealth is:¹⁵

$$H_t = \frac{Y_t}{1 - \frac{G}{R}} \tag{13}$$

¹⁵For a solution to exist, we must assume that $R > G$.

The saving rate is given by:

$$s_t = \frac{rW_{t-1} + Y_t - C_t}{rW_{t-1} + Y_t} \quad (14)$$

Suppose, for simplicity, that consumers begin life with zero assets: $W_0 = 0$. Then the saving rate in the first year of life is given by:

$$s_1 = \frac{(Y_1 - C_1)}{Y_1} = \frac{Y_1 \frac{k_1 Y_1}{1-G/R}}{Y_1} = 1 - \frac{k_1}{1-G/R} \quad (15)$$

The derivative of this expression with respect to G is unambiguously negative, because k_1 is positive and increasing G decreases the denominator of the last expression. Interpreted in terms of equation (12), the negative correlation between G and saving is due to the powerful effect of G on human wealth and therefore consumption.

After the first year of life, assets will be a function of G and of past consumption. A thorough analysis of the problem shows that among consumers with a high lifetime G , young households have a lower saving rate, but their elders have a higher saving rate, than people of the same ages with a low lifetime G . The age at which the saving rate switches from being a negative function of G to being a positive function is dependent on all the parameters of the model.¹⁶ This is one reason we restricted our household-level tests of saving behavior to young households, for whom the model's prediction of a negative derivative of saving with respect to growth is unambiguous.

4.2 *Liquidity constraints*

We will first consider a very simple model of liquidity constraints and show that it can reduce, but not eliminate, the negative influence of income growth on saving. We will then examine informally a more complex model of liquidity constraints in which forward-looking consumers must accumulate a down payment in order to purchase a house. We show that such a model at least has the potential to be consistent with our results.

4.2.1 Simple liquidity constraints. Consider a liquidity constraint of the form $W_t \geq 0 \forall t$. For simplicity we will assume that utility is of the Constant Relative Risk Aversion (CRRA) form, although the qualitative results do not depend on this assumption. For nonliquidity-constrained consumers with CRRA utility of the form $U(C) = C^{1-\rho}/(1-\rho)$, the growth rate of consumption is given by:

$$\frac{C_{t+1}}{C_t} = (\beta R)^{1/\rho} \equiv \omega \quad (16)$$

¹⁶Note, however, that the derivative of the level of assets with respect to the lifetime growth rate of income is negative for all age groups.

It is straightforward to show in this model that, for a non-liquidity-constrained consumer who starts life with zero assets, if $\omega > G$, i.e., desired consumption growth is greater than expected income growth, then consumption in the first year will be less than income. Furthermore, in all subsequent years assets will be positive. For such consumers, therefore, liquidity constraints would never bind, so the same negative relationship between saving and G derived above for the unconstrained *LC/PIH* model above will apply.

If, however, $\omega < G$, then a non-liquidity-constrained consumer with zero assets would choose to spend more than current income; assets next period would become negative. If such a consumer were liquidity-constrained, consumption would be limited to current income, and she would enter the next period with zero assets, at which point she would face exactly the same maximization problem as in the first period and would therefore again be constrained. For such consumers, assets and saving will always be zero, so the derivative of the saving rate with respect to the growth rate of income will also be zero. The “human-wealth effect” on consumption is zero because consumption is already at its maximum obtainable value.

The derivative of saving with respect to G is therefore a function of tastes, the level of G , assets, and other parameters. However, the derivative is always either zero or negative. At best, therefore, this simple model of liquidity constraints could explain empirical results in which the saving was unrelated to income growth. It cannot, however, explain our empirical result of a positive correlation.

4.2.2 Sophisticated liquidity constraints. Only a small fraction of total consumer debt in the United States is uncollateralized. Of collateralized debt, the considerable majority is for home mortgages. A more realistic description of liquidity constraints might therefore be that consumers can borrow, but only to finance the purchase of a collateralized asset. Although we have not been able to complete a formal theoretical analysis, it appears to be at least possible to generate a positive correlation between saving and growth in a model in which consumers purchase houses via mortgage borrowing. Two assumptions are important for generating such a result: first, collateralized borrowing must require households to accumulate a down payment equal to some fraction of the value of the house. Second, the desired value of the house must be a function of lifetime income.

The intuition for how such a model could generate a positive correlation between growth and saving is simple. During the first part of their lives, consumers save only in order to accumulate the down payment for their home. Holding income in the first period of life constant, the greater is G , the larger is lifetime income. The higher is lifetime income, in turn, the more expensive is the desired home, which requires a larger down payment, which necessitates more saving.

We illustrate this possibility with a very simple model, not derived from a utility maximization framework. Consider a consumer who expects his income to grow at rate $G > 1$ over his entire 50-year economic lifetime (say, ages 25 to 75). The real interest rate is assumed to be zero (that is, $R = 1$). The simplest possible assumption about consumption is that, in the absence of the home-buying motive, it would be constant over the lifetime (an optimizing consumer would chose constant consumption if $R = \beta$). If this consumer were not liquidity constrained, he would borrow when young, but simple liquidity constraints of the kind described above would cause him to set consumption equal to income over his lifetime.

Now assume that the consumer buys a home in the tenth year of life, and that the value of the home is some proportion h of lifetime income, $V = hH_1$. Assume further that he accumulates the down payment by depressing consumption by the same amount in each year of the first ten years of life. Finally, assume that, after the home is purchased, spending is elevated in each remaining year of life by a constant amount sufficient, by the last year of life, to have dissipated the wealth represented by the down payment. If the size of the down payment is given by a factor d times the value of the home, the consumer's lifetime spending pattern can be summarized by:

$$\begin{aligned} C_t &= Y_t - \frac{hdH_1}{10} & \text{if } t \leq 10 \\ &= Y_t + \frac{hdH_1}{40} & \text{if } t > 10 \end{aligned} \tag{17}$$

The household saving rate $(Y_t - C_t)/Y_t$ will therefore be given by:

$$\begin{aligned} s_t &= \frac{hdH_1}{10Y_t} & \text{if } t \leq 10 \\ &= -\frac{hdH_1}{40Y_t} & \text{if } t > 10 \end{aligned} \tag{18}$$

and assets, which were identically zero in the simple liquidity constrained case, would be given by:

$$\begin{aligned} W_t &= \frac{t}{10}hdH_1 & \text{if } t \leq 10 \\ &= \frac{50-t}{40}hdH_1 & \text{if } t > 10 \end{aligned} \tag{19}$$

Since H_1 is a positive function of G , it is clear that in this model the saving rate for young households will be a positive function of income growth, as

will assets at every age of life. Although this particular formulation of the lifetime consumption problem is highly unrealistic, and is flawed in that it is not derived from an explicit maximization problem, it illustrates at least a potential channel through which income growth might be positively related to saving. Whether a model with a realistic lifetime income process and in which the timing and the magnitude of housing purchase were derived optimally could generate a similar positive correlation is unclear, but could be a valuable path for future research.¹⁷ However it seems to us unlikely that saving for downpayments is large enough to explain the aggregate correlation between saving and growth.

4.3 *Heterogeneity in discount rates*

One intuitive explanation for the correlation between income growth and saving rates observed in household data is individual differences in the rate at which future utility is discounted. Patient individuals might be expected not only to save more, but also to be more willing to choose occupations in which income starts low but grows quickly.

This argument is not as straightforward as it appears, however. To begin with, it requires that all young households be subject to liquidity constraints (of the “simple” kind discussed above). If there were no liquidity constraints it would not be necessary for consumers entering a high-growth profession to be more willing to defer consumption. Each household could choose the profession or education that maximized lifetime earnings and could then choose the lifetime consumption profile independently of the income profile, so there would be no reason for patience to be related to income growth.

If all young households were pushing against liquidity constraints, patient consumers would be more willing to endure low consumption today in exchange for high consumption tomorrow, so there might be a correlation between income growth and patience. But there would be no variation in saving rates because they would all save zero. The correlation between saving and growth would be zero.

There is a case in which the story can be made to work. Imagine that there are two young households, one patient and one impatient, and two occupations, one with a slow-income growth path that starts out high and the other with a fast-growth path that starts out low but has a higher present discounted value. Suppose young households cannot borrow. Imagine that the impatient household is unwilling to choose the fast-growth occupation because it would have to depress consumption for too long before reaping the rewards of higher future consumption, but the patient household chooses

¹⁷Sheiner (1991) provides a complete analysis of this problem, in which the size of the house purchased and the date of purchase are endogenous.

the high-growth occupation. The patient consumer could be so patient that he saves even given his rapid-growth path, while the impatient consumer could be so impatient that he will be up against the liquidity constraint and will save nothing, even given his slow-growth path. In this case there would be a positive correlation between saving and growth. Support for such a story comes from Shapiro and Slemrod's (1993) study of the effects of the 1992 reduction in income-tax withholding, which should have only affected the consumption of liquidity-constrained households. They find that faster expected future income growth made it *less* likely that a household would report that it intended to spend the increase in its take-home pay.

Although a model like the one just described could be responsible for our household-level results, it seems inadequate as an explanation for our aggregate evidence on growth and saving. While we cannot rule out permanent differences in discount rates across countries, and are even willing to entertain the possibility of exogenous changes in discount rates within countries, we do not believe that the discount rate within a country should be a function of lagged aggregate income growth. Yet in our macro data we find that within countries increases in growth lead to increases in saving. To explain this correlation as resulting from discount rates, one would have to postulate that increases in growth lead to decreases in discount rates. We do not find this plausible.

4.4 *Precautionary saving*

All the foregoing analysis was conducted assuming that the future path of income is known with certainty. However, a growing body of recent research has argued that income uncertainty has profound consequences for the qualitative and quantitative predictions of consumption models. One intuitive result from that literature is that if consumers have a precautionary saving motive, they will be more reluctant to spend out of uncertain future income than out of certain current income (see, e.g., Barsky, Mankiw, and Zeldes, 1986). This should reduce the magnitude of the human-wealth effect on current consumption.

More is required to explain our empirical results than a reduction of the negative effect of human wealth on saving, however: there must also be some reason for a positive effect of growth on saving. The buffer-stock model of saving developed by Deaton (1991) and Carroll (1992a,b) is promising in this regard. Carroll (1992b) solves a model similar to that of equation (11) except that the income process is described as follows:

$$P_t = GP_{t-1}N_t \tag{20}$$

$$Y_t = P_tV_t \tag{21}$$

N_t is a lognormally distributed white noise error term, so that P (permanent income) evolves according to a random walk with drift. Income Y is given by P multiplied by a transitory shock V_t . V_t is a mixture of two distributions: with a small probability V_t is equal to zero (representing periods of unemployment), but if V_t is not equal to zero it is lognormally distributed white noise. Carroll (1992b) shows that in this model consumers who are sufficiently impatient will have a target wealth-to-income ratio w^* towards which their wealth will converge.¹⁸ At the target wealth/income ratio the personal saving rate will be given by:

$$s_t = gw^* \quad (22)$$

The derivative of the saving rate with respect to the growth rate of income is therefore:

$$\frac{ds_t}{dg} = w^* + g \frac{dw^*}{dg} \quad (23)$$

Unfortunately it is not possible to derive an analytical expression for w^* , so this equation cannot be signed analytically. However, Carroll (1992b) reports simulations of the model over a range of values for g , and finds that the relationship between saving and growth is strongly positive for the parameter values he uses.

The intuition for the positive association between saving and growth is simple. If consumers desire to hold a fixed target wealth/income ratio, then if income is growing faster, wealth must grow faster. To make wealth grow faster it is necessary to save more. An offsetting effect is that the target wealth/income ratio is lower when income growth is higher (i.e., $dw^*/dg < 0$). This is the human-wealth effect on saving, and as above it is negative. The simulations in Carroll (1992b) found, however, that dw^*/dg was quite small. The human-wealth effect is diminished in this model because households are reluctant to consume today out of expected future income if that future income is uncertain.

Unfortunately, even this model is not fully consistent with our empirical results. Recall that the regressions in the PSID and SCF were not of saving rates on growth rates, but rather were of wealth-to-income ratios on growth rates. As noted in the previous paragraph, even in the buffer-stock model the wealth-to-income ratio should be negatively related to growth. Thus, the buffer-stock model is consistent with the qualitative result that the saving rate is positively correlated with the growth rate of income, but not with our actual empirical result that the wealth/income ratio is positively correlated with income growth.

¹⁸Consumers who are not impatient will accumulate assets indefinitely. As assets grow large relative to income uncertainty, uncertainty becomes less and less important, and in the limit there is no difference between consumption in this model and consumption in a model without uncertainty.

4.5 Habit formation

Again, consider a utility-maximizing household, but suppose now that utility is a function of the excess of consumption over some habit stock carried over from the past. The simplest such framework is one in which utility is given by $U(C_t - aC_{t-1})$. Muellbauer (1988) shows that if the utility function is homothetic, consumption in period t will be given by:

$$C_t = k_t \frac{(1+r)W_{t-1} + H_t}{\phi_t} + aC_{t-1}(1 - k_t) \quad (24)$$

where $k_t > 0, \phi_t > 1$ are not functions of G . Suppose the consumer begins life with a habit stock C_0 equal to initial labor income Y_0 . Assume again that initial assets W_0 are zero. Assume, also, that income in the first period of life is given by $Y_1 = \Gamma Y_0$, but income growth is constant at rate G thereafter. Then the saving rate in the first period of life will be given by:

$$\begin{aligned} s_1 &= \frac{Y_1 - C_1}{Y_1} \\ &= \frac{Y_1 - \frac{k_1 \frac{Y_1}{1-G/R}}{\phi_1} - aY_0(1 - k_1)}{Y_1} \\ &= 1 - \frac{k_1}{(1 - G/R)\phi_1} - \frac{a(1 - k_1)}{\Gamma} \end{aligned} \quad (25)$$

Using this formula it is possible to analyze the effect on saving of two separate growth experiments: increasing the rate of growth in the first period of life, Γ , or increasing the rate of growth over the rest of the lifetime, G . It turns out that the two experiments have opposite effects: $ds_1/d\Gamma$ is positive, but ds_1/dG is negative.

The intuition behind each of these effects is simple. The second captures the positive influence of human wealth on current consumption, just as in the standard LC/PIH model above. The first captures the fact that, given a previous habit stock, consumption will adjust upward only sluggishly in response to an increase in income. Increasing Γ raises income in the first period more than consumption, and therefore increases the first-period saving rate. We call this the habit-stock effect.

If $\Gamma = G$ in the formula above, then under any reasonable parameter values the total derivative of the saving rate with respect to G is negative, because the human wealth effect overpowers the habit-stock effect. The assumption that income grows at one rate in the first year of life and another rate thereafter is clearly artificial, and was designed to provide a tractable analytical example of how habit formation could at least potentially cause a positive correlation between short-term growth and saving. However, an

income profile in which income grows at one rate during the early part of a career and a different, lower, rate for the remainder is quite plausible. In fact, such a pattern is a good qualitative description of our own data on income growth from our household data sources.¹⁹ Rapid income growth in the near future followed by slower growth in the far future may also be a good description of the experience of households in rapidly growing economies, such as the high-saving East Asian countries examined earlier. In such cases, the sign of the correlation between saving and growth will depend on the relative strengths of the human-wealth effect and the habit-stock effect.

The analytical formulas for the saving rate at ages beyond the first year of life are forbiddingly complex, so we resorted to simulations in order to explore the model's predictions for the growth/saving relationship. The first simulation assumes a utility function of the form $U(C_t - aC_{t-1}) = (C_t - aC_{t-1})^{1-\rho}/(1-\rho)$, where ρ was set at 4. The income growth factor is Γ for the first 10 years of life and $G = 1$ for the remainder of an infinite horizon. We assumed an interest rate equal to the discount rate at 3 percent. Our experiment was to compare saving-rate profiles for the first part of life when $\Gamma = 1.01$ and when $\Gamma = 1.06$.

Previous work has argued that habit formation may be able to explain excess smoothness in aggregate consumption (Deaton, 1987) or the equity premium puzzle (Constantinides, 1990) if the habit-formation parameter a is at least .8. Our first simulations therefore assume a value of $a = .8$. The results are presented in Figure 10 a and b. The top panel shows the path of income (solid lines) and consumption (dashed lines) under the two growth assumptions. The bottom panel shows the path of the saving rate for the low-growth (solid line) and high-growth (dashed line) cases.

The bottom panel shows that, for these parameter values, during the first ten years of life the average saving rate is lower for the fast-growing household than for the slow-growing household – the human-wealth effect outweighs the habit-stock effect. Nevertheless, the results represent some progress relative to the LC/PIH model, because the negative effect of growth on saving is much smaller than in that model.

The next set of simulations, in Figures 11 a and b, repeats the previous experiment but with a habit-formation parameter of $a = .9$. This increase in the strength of habit formation is sufficient to retard consumption enough to generate a higher average saving rate for the high-growth consumer than for the low-growth consumer in the first ten years of life, although saving is still lower in the first three or four years. A further boost in the habit-formation parameter to $a = .95$ (not shown) guarantees that the high-growth household

¹⁹In the 1961–62 CEX, for instance, we found wide dispersion in growth rates by occupation or educational category for young households, but much less dispersion (and lower average growth) for older households.

Figure 10a
Income and Consumption
Habit Parameter $a = .8$

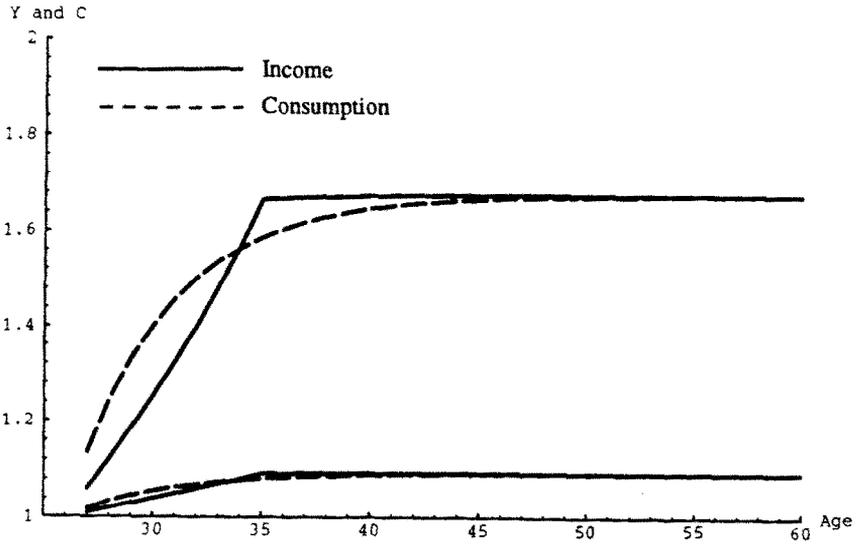


Figure 10b
Saving Rate
Habit Parameter $a = .8$

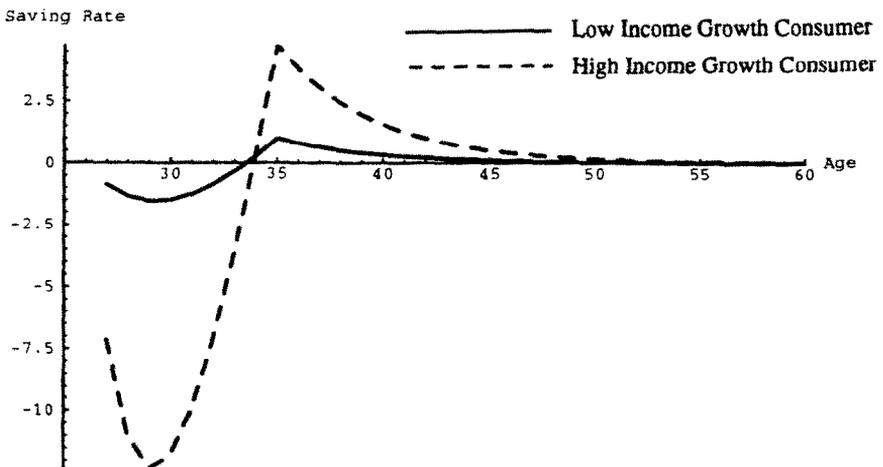


Figure 11a
Income and Consumption
Habit Parameter $a = .9$

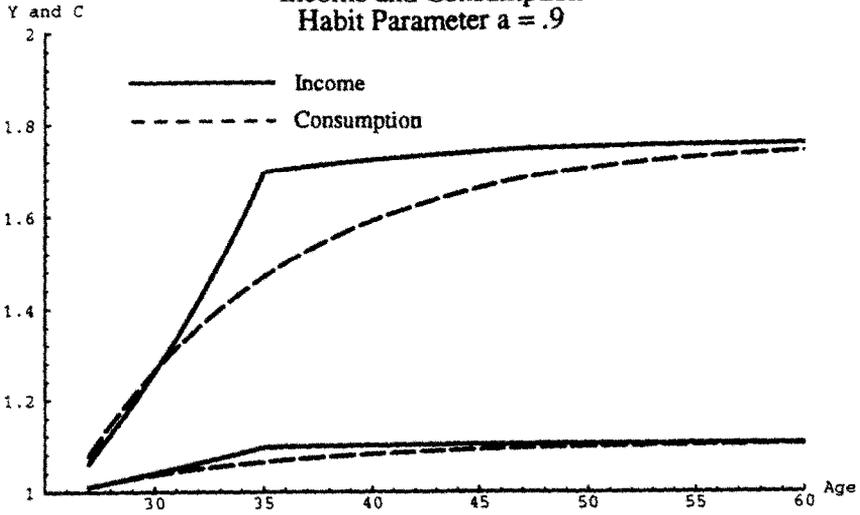
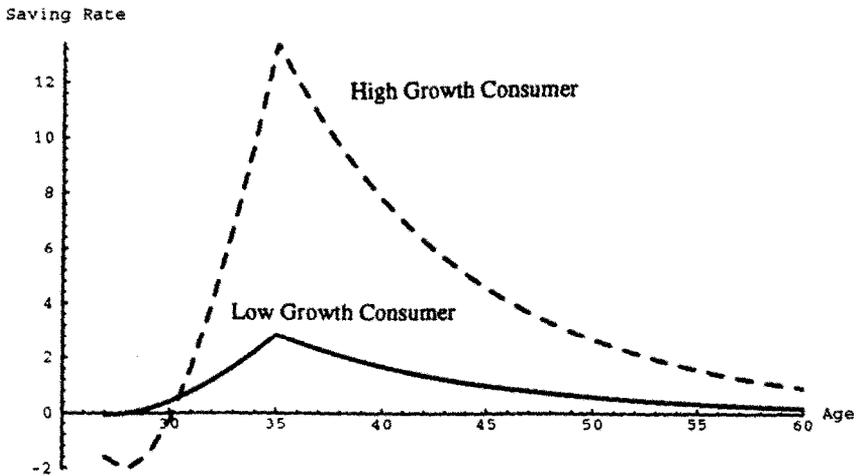


Figure 11b
Saving Rate
Habit Parameter $a = .9$



has higher saving in every year of the first ten years of life.

A value of $a = .9$ or $.95$ implies that consumers care enormously about how their current consumption compares to their previous consumption, but care very little about the absolute level of their consumption. Although our intuition is not strong for what values are plausible, a value of $.9$ seems uncomfortably high. We know of little direct empirical evidence on the value of a other than a paper by Dynan (1992) using food consumption data from the PSID. She finds an upper bound near $.7$ for a .

We should note that the shorter is the period of rapid growth at the beginning of life, the weaker will be the human-wealth effect on consumption, and thus the easier it will be for the habit-stock effect to outweigh the human-wealth effect. The limit, of course, is the case presented analytically above, where income grows rapidly only in the first year of life. If income grows rapidly for the first 20 (rather than 10) years of life, even with $a = .95$ the correlation between saving and growth is negative.

Our conclusion is that the simple habit-formation model can theoretically explain our empirical results, but only if we make rather implausible parameter assumptions. The problem is that the human-wealth effect is tremendously strong and tends to overpower the habit-stock effect unless habits are also very powerful.

4.6 *Combining models*

In our discussion of liquidity-constrained and precautionary-saving models we concluded that, although both were able to reduce the human-wealth effect on consumption, neither provided a mechanism for producing a sufficiently strong positive correlation between saving and growth. The problem with the habit-formation model was the opposite: a low initial habit stock can justify a positive association between saving and growth, but for reasonable parameter values the powerful human-wealth effect overwhelms the habit-stock effect. It is tempting, therefore, to speculate about whether combining the habit-formation model with one of the other two models could produce a fully satisfactory explanation of our puzzle. Unfortunately, analyzing such models formally is beyond the scope of this paper, but we wish to indulge now in some brief speculation about the likely results from such hybrid models.

We do not believe that adding liquidity constraints to the habit-formation model would solve the puzzle. As in the analysis of the standard LC/PIH model above, it should be possible to split the population into those who are constrained and those who are unconstrained. For the unconstrained, the human-wealth effect on consumption would be undiminished. For the constrained, saving would again be identically zero. The qualitative result should be the same as in the LC/PIH model: the negative response of saving to growth would be lessened but not reversed.

We have more hope for the prospects when uncertainty is added to the habit-formation model. As in the usual LC/PIH framework, uncertainty about future income should significantly reduce the willingness of households to base current spending on expected future income. Uncertainty should also make households more reluctant to consume today for fear of creating a habit stock which might prove impossible to maintain in the event of a bad income shock tomorrow. Increasing the expected growth rate of income while leaving intact the possibility of big drops might therefore produce little effect on current consumption. Given the current habit stock, however, the optimality of slow adjustment of consumption to income should remain, leaving the positive correlation between growth and saving intact. We conclude that a model with both habit formation and income uncertainty may hold out the best hope for explaining our micro results. To the best of our knowledge, no such models have been formally analyzed in the literature to date. Such an analysis would be a valuable contribution.

Another possible way to reduce the human-wealth effect is to add bequests to the model. If habit formation is strong, a faster growth rate of income might result mainly in a larger bequest to one's offspring rather than in an increase in consumption during one's own lifetime.

5 Conclusions

We believe that we have established two interesting new empirical facts. First, at the aggregate level, periods of high income growth appear to be followed by periods of high saving. Second, among young households, those households who should expect faster income growth appear to save more than households who should expect slower income growth.

Although it is possible that these two findings have entirely different explanations, a common model which could explain both results is highly desirable. We therefore considered whether the most common modifications of the standard LC/PIH model, including versions incorporating liquidity constraints, precautionary saving, and habit formation, were capable of reproducing the observed positive correlation between anticipated growth and saving across households. Although some version of each of the models was capable of producing a positive correlation between saving and growth, none of the models was fully satisfactory. We speculated, however, that a model which combines habit formation and income uncertainty may provide the best hope for explaining our empirical results.

Returning to the topic of modelling economic growth, the positive effect of income growth on saving has implications both for estimating models and for the dynamic response of an economy to shocks. In the limit, one could argue that the common conclusion that raising a country's saving rate is a

good way to raise its growth rate is simply wrong. We do not take this view. But the endogeneity of saving with respect to growth suggests that the estimated effects of saving on growth may be overstated. In terms of work (such as Mankiw, Romer, and Weil, 1992) that uses cross-country variation in saving rates and growth rates in order to estimate parameters of the production function, this endogeneity problem means that the contribution of capital to output (the exponent on capital in a Cobb-Douglas production function) may be overstated. Without a proper instrument for the saving rate, it is impossible to estimate the true structural effect of saving on growth. Unfortunately, we do not know of an eligible instrument. However, recognizing the endogeneity of saving leads us to moderate the policy recommendations that come from current growth models.

Recognizing the effects of growth on saving also leads to a wide range of possible dynamics of growth models. To take one example, a transitory, negative shock to growth may be propagated via a response of saving: lower growth lowers the saving rate, further lowering growth. Such a description has something of a ring of truth in describing the OECD in the two decades since the OPEC shock.

Technical Appendix

This appendix describes the Two Sample Two Stage Least Squares (TS2SLS) estimation procedure used for the SCF estimates and the pooled SCF/PSID estimates in Table 5. Assume an underlying population in which each member i is characterized by values for X_i, Y_i , and Z_i . Two samples $h = \{1, 2\}$ are drawn from this population; the first sample (corresponding to the PSID in our empirical work) contains observations on X, Y , and Z , but the second sample (corresponding to the SCF) contains observations only on X and Z . The goal is to estimate δ in an equation of the form:

$$Y_h = X_h \delta + e_h \quad (\text{A.1})$$

in each of the samples. The usual Two-Stage Least Squares (2SLS) procedure for estimating this equation in the first data set involves estimating the equation:

$$X_1 = Z_1 \alpha + u \quad (\text{A.2})$$

Under assumptions described below, a consistent estimator for α is given by:

$$a_1 = (Z_1' Z_1)^{-1} Z_1' X_1$$

The 2SLS estimation is performed by constructing $\hat{X}_1 = Z_1 a_1$ and estimating the equation:

$$Y_1 = \hat{X}_1 \delta + u_1 \quad (\text{A.3})$$

via OLS. The usual set of assumptions under which 2SLS estimation produces a consistent estimate of δ is given by:

$$\text{plim}(Z_h' X_h / n_h) = \Sigma_{zx} \quad (\text{A.4})$$

$$\text{plim}(Z_h' e_h / n_h) = 0$$

$$\text{plim}(Z_h' u_h / n_h) = 0$$

Consistency is proven as follows. Define $\hat{v}_1 = X_1 - \hat{X}_1$. Then:

$$Y_1 = \hat{X}_1 \delta + u_1 = \hat{X}_1 \delta + (e_1 + \hat{v}_1 \delta)$$

Estimating equation (A.3) by OLS yields d_1 :

$$\begin{aligned} d_1 &= (\hat{X}_1' \hat{X}_1)^{-1} \hat{X}_1' Y_1 = (\hat{X}_1' \hat{X}_1)^{-1} \hat{X}_1' (\hat{X}_1 \delta + u_1) \\ &= \delta + (\hat{X}_1' \hat{X}_1)^{-1} \hat{X}_1' u_1 \\ &= \delta + (\hat{X}_1' \hat{X}_1)^{-1} \hat{X}_1' (e_1 + \hat{v}_1 \delta) \\ &= \delta + (\hat{X}_1' \hat{X}_1)^{-1} \hat{X}_1' e_1 \end{aligned} \quad (\text{26})$$

where the last equality follows because $\hat{X}'_1 \hat{v}_1 = 0$ by construction. d_1 is a consistent estimator for δ because:

$$\begin{aligned} plim(d_1 - \delta) &= plim(\hat{X}'_1 \hat{X}_1/n_1)^{-1} plim(\hat{X}'_1 e_1/n_1) \\ &= \Omega^{-1} 0 \end{aligned}$$

where the fact that Ω exists and is a finite matrix follows from assumptions (A.4) and $plim(\hat{X}'_1 e_1/n_1) = 0$ follows from $plim(Z'_h e_h/n_h) = 0$.

Now consider constructing $\hat{X}_2 = Z_2 a_1$ in the second data set. OLS estimation of the equation $Y_2 = \hat{X}_2 \delta + u$ yields:

$$d_2 = \delta + (\hat{X}'_2 \hat{X}_2)^{-1} \hat{X}'_2 (e_2 + \hat{v}_2 \delta)$$

and thus

$$\begin{aligned} plim(d_2 - \delta) &= plim(\hat{X}'_2 \hat{X}_2/n_2)^{-1} \{plim(\hat{X}'_2 e_2/n_2) + plim(\hat{X}'_2 \hat{v}_2 \delta/n_2)\} \\ &= plim(\hat{X}'_2 \hat{X}_2/n_2)^{-1} plim(\hat{X}'_2 e_2/n_2) \\ &= \Omega^{-1} 0 \end{aligned}$$

Thus the TS2SLS procedure of estimating the first-stage equation in the first data set and the second-stage regression in the second data set generates a consistent estimate of δ .

The asymptotic variance-covariance matrix of the 2SLS estimator is given by:

$$plim(d_h - \delta)(d_h - \delta)' = \Omega^{-1} plim(\hat{X}'_h e_h e'_h \hat{X}_h/n_h) \Omega^{-1}$$

In the usual 2SLS estimation we observe $\hat{e}_h = Y_h - X_h d_h$. If the errors e_h are homoskedastic, a consistent estimator of $plim(\hat{X}'_h e_h e'_h \hat{X}_h/n_h)$ is given by $s^2_{e,h}/n_h \Omega$ where $s^2_{e,h} = \hat{e}'_h \hat{e}_h/n_h$. However, in the second data set for TS2SLS we do not observe X so we cannot construct \hat{e}_h . Note, however, that:

$$\begin{aligned} ee' &= (u - v\delta)(u - v\delta)' \\ &= uu' - v\delta u' - (v\delta)'u + (v\delta)(v\delta)' \end{aligned}$$

Assume that all of these terms are homoskedastic, so that for any household i in either data set, $E[u^2] = \sigma_u^2$, $E[v_i \delta u_i] = \sigma_c$ and $E[(v_i \delta)^2] = \sigma_{v\delta}^2$, where σ_c is the covariance between $v_i \delta$ and u_i . The asymptotic variance-covariance matrix for d_h is therefore given by:

$$\begin{aligned} plim(d_h - \delta)(d_h - \delta)' &= \Omega^{-1} (\sigma_u^2 - 2\sigma_c + \sigma_{v\delta}^2) \Omega \Omega^{-1} \\ &= (\sigma_u^2 - 2\sigma_c + \sigma_{v\delta}^2) \Omega^{-1} \end{aligned}$$

If X, Y , and Z were all observed in data set h we could construct:

$$\begin{aligned} s_{u,h}^2 &= \hat{u}'_h \hat{u}_h / n_h \\ s_{c,h} &= \hat{u}'_h \hat{v}_h d_h / n_h \\ s_{v\delta,h}^2 &= (\hat{v}_h d_h)' (\hat{v}_h d_h) / n_h \end{aligned}$$

and standard proofs demonstrate that $plim s_{u,h}^2 = \sigma_u^2$, $plim s_{c,h} = \sigma_c$, and $plim s_{v\delta,h}^2 = \sigma_{v\delta}^2$. All three of these terms can be computed in the first data set, but only the first can be computed in the second data set (where X is not observed so \hat{v} cannot be computed). Suppose, however, that the number of observations in the first data set is a function of the number in the second data set, $n_1 = kn_2$. Then define:

$$\begin{aligned} s_{e,2}^2[n_2] &= \hat{u}'_2 \hat{u}_2 / n_2 - 2\hat{u}'_1 \hat{v}_1 d_1 / kn_2 + (\hat{v}_1 d_1)' (\hat{v}_1 d_1) / kn_2 \\ plim s_{e,2}^2[n_2] &= \sigma_u^2 - 2\sigma_c + \sigma_{v\delta}^2 \end{aligned}$$

A consistent estimate of the variance-covariance matrix is then given by:

$$n_2^{-1} s_{e,2}^2[n_2] (\hat{X}'_2 \hat{X}_2 / n_2)^{-1}$$

because

$$plim [s_{e,2}^2[n_2] (\hat{X}'_2 \hat{X}_2 / n_2)^{-1}] = (\sigma_u^2 - 2\sigma_c + \sigma_{v\delta}^2) \Omega^{-1}.$$

In practice, the estimate of $s_{e,2}^2$ that we use is given by:

$$s_{e,2}^2 = s_{u,2}^2 - 2s_{c,1} + s_{v\delta,1}^2$$

Pooled estimation using all the data from both data sets is only trivially more complex. Consider constructing a third data set $h = 3$ by stacking the values of Y and \hat{X} :

$$Y_3 = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \quad \hat{X}_3 = \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \end{bmatrix}$$

Then estimation of d_3 proceeds exactly in parallel with estimation of d_2 as described above, substituting the subscript 3 for 2. The consistent estimate of the variance of the error is given by:

$$s_{e,3}^2 = s_{u,3}^2 - 2s_{c,1} + s_{v\delta,1}^2$$

The assumption of homoskedasticity is not essential. It is relatively straightforward to allow for heteroskedasticity of unknown form in the u 's. In practice the empirical results were not much different when such heteroskedasticity was allowed. Furthermore, the heteroskedasticity tests reported in Table 5 rarely rejected the null of homoskedasticity, so we do not

report the derivation or results for the heteroskedasticity-robust form of the test.

The test of overidentifying assumptions reported in Table 5 is based on Hansen's (1982) proof that the statistic $\hat{e}'Z(Z'\hat{e}\hat{e}'Z)^{-1}Z'\hat{e}$ should be distributed χ^2 with degrees of freedom equal to the number of overidentifying restrictions. Although this statistic cannot be directly computed in the second sample because X_2 is not observed (and so \hat{e} is not observed), it should be asymptotically valid to substitute \hat{u} for \hat{e} in the statistic because $plim(Z'\hat{u}\delta/n) = 0$. This is the test whose p -value we report for the SCF and pooled regressions.

A note is in order about the relationship of this procedure to the Two Sample Instrumental Variables (TSIV) procedure of Angrist and Krueger (1990). Angrist and Krueger state that their procedure can be performed on any two data sets such that, in one, only Y and Z are observed, and in the other only X and Y are observed. While it is true that their TSIV procedure produces consistent estimates of the coefficient vector δ , we believe that the variance-covariance matrix they propose is valid only if $plim(X'e/n) = 0$, or, equivalently in the notation above, if $\sigma_c = 0$. (This corresponds to their assumption A2(i).) But this is the case where simple OLS estimation would be consistent in a dataset which contained both X and Y . Thus we believe that their TSIV estimator does not allow for valid hypothesis testing in the usual instrumental variables case where there is simultaneity between X and Y .

Data Appendix

This appendix describes the aggregate and the household-level data sources and variables constructed using them.

The Summers-Heston Data Set

Saving was constructed as follows:

$$\text{sav} = 1 - (c * pc * pcus + g * pg * pgus) / (p * 100)$$

Where the variables are

- c = Real consumption
(% of RGDP, 1985 international prices)
- g = Real public consumption
(% of RGDP, 1985 international prices)
- pc = Price level of consumption
(100* PPP of consumption/Exchange Rate)
- pg = Price level of public consumption
(100* PPP of government/Exchange Rate)
- p = Price level of GDP
(100*PPP of GDP/Exchange Rate)
- pcus = Price level of consumption in the U.S.
- pgus = Price level of government in the U.S.

The first five variables listed (c, g, pc, pg, and p) are taken directly from the published data. The other two variables, which are part of the data underlying the published tables, were supplied by Dan Nuxoll of the International Comparison Project.

The Panel Study of Income Dynamics

The PSID data were taken from Wave XX of the study. The income variable used was pretax noncapital household income, constructed by subtracting capital income from total household income. Capital income in the years 1981–1987 in the PSID consists of the sum of interest, dividends, and rent for all members of the household plus the asset portion of business, farming, and gardening income. The income data were deflated using the PCE deflator with a base year of 1982. Our measure of net worth was the sum of all assets minus the sum of all debt reported in the 1984 wealth survey.

Wave XX of the PSID dataset contains data for 8,129 individuals who were ever heads of households. The sample was restricted to households which fulfilled the following restrictions: The individual in question was head of the household in 1981 and 1987, and there was no change in marital status between 1981 and 1987. The head was aged 30–39 in 1984. Valid data were

reported for occupation and education. The head was employed for the whole year in both 1981 and 1987, and was never self-employed or a farmer. Educational status did not change between 1981 and 1987. Enough valid wealth data existed to create a measure of net worth. The household had never inherited anything. The household did not report exactly zero wealth in 1984.

The 3-digit occupation code was compressed into six occupation categories²⁰ in order to provide a small set of occupation dummies that would be compatible with the SCF data. The eight education categories reported in the PSID were compressed to six, also for compatibility with the SCF.

The Survey of Consumer Finances, 1983

The 1983 SCF contains data on 4,303 households. The income variable we used was labor income of the head of household. The wealth variable was total household net worth at the end of 1982. The occupation and education dummy variables were constructed from the occupation and education codes contained in the survey in order to be consistent with the definitions of occupational and education groups used in the PSID.

Our sample was restricted to households for whom the following conditions held. The household head was age 30–39 and was married. Valid data were reported for the occupation and education of the head, and the occupation was neither self-employment nor farming. The head reported positive wage and salary income in 1982, and positive wealth holdings at the end of 1982.

The 1961–62 Consumer Expenditure Survey

The 1961–62 CEX contains data on 13,728 households. For estimating the wage equation (10), our definition of income was total household noncapital income, given by total household income minus interest, dividend, and rent income. Our definition of the personal saving rate was the log of total disposable income (total income minus total taxes) minus the log of total consumption expenditures. Consumption expenditures in the CEX include direct out-of-pocket expenditures for durable goods, including cars. If the car is purchased with a loan, however, in the year of purchase only the down payment and any loan payments would appear as expenditures.²¹

²⁰The exact mapping is available upon request.

²¹We repeated the regressions of Table 6, but excluding households who made car purchases and found similar results, although with somewhat greater statistical significance than the reported regressions.

Households which met the following criteria were included in the estimation of equation (10). Valid occupation and education data were reported by the head, who was neither a farmer nor self-employed. There was no change in family structure over the course of the year. The head worked full time. The race of the head was white. Household noncapital income was above the 5th percentile in the income distribution. The regressions in Table 6 were subject to two further restrictions: The head of household was aged 30–39, and consumption was above the 5th percentile in the consumption distribution. The distributional restrictions for income and consumption were imposed because otherwise a few outliers with implausibly low consumption or income unduly influenced the results.

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