APMA 1210 – HOMEWORK 1

Reading: Sections 1.1 and 1.2 of the textbook

Problem 1: (5 points)

Dr. Peyam has an awesome T-shirt store called Teespring. It has contracted with a manufacturing company to produce two types of T-Shirts: "Use the Chain Rule" and "Körper." The major steps in the manufacturing of these shirts are: cutting the material, sewing, and decorating. The table below shows the resource requirements for each type of shirt and total availability of resources, all measured in hours.

T-Shirt	Cutting (h)	Sewing (h)	Decoration (h)	Profit ($)
Chain Rule	0.2	0.5	0.42	20
Körper	0.15	0.3	0.15	12
Resource Avail	50.00	60.00	55.00	

Set up a linear programming model (decision variables,constraints, objective function) to determine how many "Use the Chain Rule" and how many "Körper" the firm should produce in order to maximize profit. Do **not** solve the model.

(Turn Page)

Date: Due: Wednesday, September 21, 2022 at 11:59 pm.

Problem 2: (8 points, 2 points each)

For each Linear Programming (LP) problem below, convert the problem to standard form and identify which of Cases 1-4 apply:

- Case 1: The LP has a unique optimal solution.
- Case 2: The LP has two or more solutions
- Case 3: The LP has no solution
- Case 4: The LP is unbounded: there are points in the feasible region with arbitrary large Z-values (for a max problem) or arbitrary small Z (for a min problem).

▶ (LP 1):

$$\begin{aligned} \text{maximize } & z = x_1 + x_2 \\ \text{subject to } & x_1 + x_2 \leq -2 \\ & 4x_1 - x_2 \geq 5 \\ & x_1, x_2 \geq 0 \end{aligned}$$

▶ (LP 2):

$$\begin{aligned} \text{maximize } & z = -x_1 + 3x_2 \\ \text{subject to } & x_1 - x_2 \leq 4 \\ & x_1 + 2x_2 \geq 4 \\ & x_2 \geq 0 \end{aligned}$$

- (LP 3):

$$\begin{aligned} \text{maximize } & z = 4x_1 + 6x_2 \\ \text{subject to } & -x_1 + x_2 \leq 11 \\ & x_1 + x_2 \leq 27 \\ & 2x_1 + 5x_2 \leq 90 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- One of the four cases is not covered by (LP1)−(LP3). Construct an example of that case.

Problem 3: (7 points) This is Exercise 1.9 in the textbook

Consider a school district with I neighborhoods, J schools, and G grades at each school.

Each school j has a capacity of c_{jg} for grade g. In each neighborhood i, the student population of grade g is s_{ig}. Finally, the distance of school j from neighborhood i is d_{ij}.

Formulate a linear programming problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students.

(You may ignore the fact that numbers of students must be integer)