APMA 1210 – HOMEWORK 2

Reading: Sections $2.1-2.2$, $2.5-2.6$, $3.1-3.2$ of the textbook

Problem 1: (6 points)

Suppose you have a portfolio of n different stocks. You bought s_i shares of stock i at price p_i. The current price of stock i is q_i, and you expect the price of stock i to be r_i next year. If you sell shares, there is a commission at a rate of 2%. In addition, there is a 25% capital gains tax on any increase in price (decreases in price are not taxed)

For example, if you buy $2,000$ shares of Gamestop for $30 per share and sell it for $40 per share, you owe $(25\%) \times 2000 \times 10 = \$5,000$ in capital gains tax and $(2\%) \times 2000 \times 40 = \$1600$ in commission.

Your net profit is the profit minus the costs. In this example, here your net profit is $\$20,000 - \$5,000 - \$1600 = \$13,400$

Formulate the following problem as an LP:

How many shares of each stock should you sell in order to raise at least K dollars in net profits while maximizing the expected value of your portfolio next year?

Note: You need to consider two types of stocks: Those where $q_i \geq p_i$ (gain) and those where $q_i < p_i$ (loss)

Problem 2: (4 points) (Problem 2.2 of the Book)

Definition: $f : \mathbb{R}^n \to \mathbb{R}$ is **convex** if for every $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and every $0 \leq \lambda \leq 1$, we have

$$f(\lambda \mathbf{x} + (1 - \lambda)\mathbf{y}) \leq \lambda f(\mathbf{x}) + (1 - \lambda)f(\mathbf{y})$$

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function and let c be a constant. Show that the following set S is convex:

$$S = \{\mathbf{x} \in \mathbb{R}^n \,|\, f(\mathbf{x}) \leq c\}$$

Problem 3: (6 points) Work through the simplex method step-by-step to solve the following LP

$$\begin{aligned}
\text{maximize } z &= x_1 + 2x_2 + 2x_3 \\
\text{subject to } 5x_1 + 2x_2 + 3x_3 &\leq 15 \\
x_1 + 4x_2 + 2x_3 &\leq 12 \\
2x_1 + x_3 &\leq 8 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

Problem 4: (4 points) Download Matlab from this website. During installation be sure to get the Optimization Toolbox by checking the corresponding box when prompted (product selection). Install and make sure you can run the program.

Then use the $A\backslash b$ function in MATLAB (here is a guide) to solve

$$\begin{cases} x_1 + 2x_2 = 1 \\ 3x_1 + 4x_2 = 2 \end{cases}$$

Please take a screenshot of your code and the result and add it to your homework. There are pdf mergers online for free. This is just to check that you actually downloaded MATLAB.