APMA 1210 – HOMEWORK 3

Reading: Sections $3.1 - 3.2, 3.4 - 3.5, 3.7, 4.1 - 4.3$ of the textbook

Problem 1: (4 points)

Use the linprog feature in MATLAB to find the optimal value and an optimal vertex of the following LP problem

$$\begin{aligned} \max z =& 2x_1 + 3x_2 + x_4 \\ \text{Subject to } & x_1 + x_2 + x_3 + x_4 \leq 2 \\ & 2x_1 + 3x_3 + 4x_4 \leq 2 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

As usual, take a screenshot of your code and the outputs. Do any modifications you need to do to z by hand, or include in the code.

From now on, feel free to use MATLAB to solve linear programming problems!

Optional: If you'd like to play around with this a bit, try to solve the LP problems in the previous two homeworks using MATLAB.

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Date: Due: Thursday, October 6, 2022 at 11:59 pm.

Problem 2: (6 points, 2 points each)

(a) Use linear programming to find one vertex of the following LP, so that one could start the simplex method.

$$\begin{aligned} \max z &= x_1 + x_2 \\ \text{subject to } 3x_1 - 4x_2 &\leq 3 \\ -x_1 - 4x_2 &\leq -7 \\ 2x_1 + 6x_2 &\leq 20 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(Please proceed the way we've done in lecture, despite the fact that -7 is negative)

(b) Draw the feasible region of the original LP. Mark the vertex found in part (a).

(c) Solve the original LP. Draw a path that the simplex algorithm could take from the vertex found in part (a) to a vertex that achieves the optimal solution.

Problem 3: (6 points, 2 points each)

(a) Write out the dual to the LP in the problem 2

(b) Solve the dual LP. What is the optimal value of the dual?

(c) Change one of the constraints in the LP in problem 2 so that the optimal value becomes unbounded. Write out the dual to the new LP. Is that dual feasible?

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Problem 4: (4 points, Problem 3.1 of the book)

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function

Suppose f has a local minimum at x, meaning there exists some $\epsilon > 0$ such that for all y with $|x - y| \leq \epsilon$ we have $f(x) \leq f(y)$.

Prove that f has a global minimum at x, meaning that for all $y \in \mathbb{R}^n$ we have $f(x) \leq f(y)$

Hint: Given $y \in \mathbb{R}^n$, first find λ (depending on x and y) small enough such that $z = (1-\lambda)x+\lambda y$ satisfies $|z - x| \leq \epsilon$. Then use the definition of local min, as well as convexity of f.