APMA 1210 Recitation 1

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1 Questions

1.1 Rabbit Food

Your beloved pet rabbit needs a special diet. Your vet says to feed your rabbit at least 24g of fat, 4g of protein, and 36g of carbohydrates every day. However, the vet also warns you not to feed your rabbit more than 5oz of food in total each day.

On your college-student budget, you can reliably afford two brands of rabbit food: E-Z-Feed and Bargain Bunny. E-Z-Feed costs $0.20/oz, and each ounce contains 12g of fat, 1g of protein, and 12g of carbohydrates. Bargain Bunny costs $0.30/oz, and each ounce contains 8g of fat, 2g of protein, and 12g of carbohydrates. Your goal is to come up with a mixture of these brands that will minimize the amount you have to spend on food while meeting your pet's needs.

(a) Write a linear program representing this problem. Clearly specify your decision variables.

(b) Reformulate your program in standard and matrix form.

(c) Draw the feasible region for the problem and clearly identify all constraints.

(d) Solve the problem.

1.2 Camera Manufacturing

This is problem #9 from Chapter 1 in *Applied Mathematical Programming*. The Candid Camera Company manufactures three lines of cameras: the Cub, the Quickiematic and the VIP, whose contributions are $3, $9, and $25, respectively. The distribution center requires that at least 250 Cubs, 375 Quickiematics, and 150 VIPs be produced each week.

Each camera requires a certain amount of time in order to: (1) manufacture the body parts; (2) assemble the parts (lenses are purchased from outside sources and can be ignored in the production scheduling decision); and (3) inspect, test,

	City	Trip cost*	Trip revenue	Average flying time (hours)
B707	A	$ 6,000	$ 5,000	1
	B	7,000	7,000	2
	C	8,000	10,000	5
	D	10,000	18,000	10
Electra	A	$ 1,000	$ 3,000	2
	B	2,000	4,000	4
	C	4,000	6,000	8
	D	——	——	20
DC9	A	$ 2,000	$ 4,000	1
	B	3,500	5,500	2
	C	6,000	8,000	6
	D	10,000	14,000	12

* Data is for a round trip.

Figure 1: Data for Airline Optimization Problem

and package the final product. The Cub takes 0.1 hours to manufacture, 0.2 hours to assemble, and 0.1 hours to inspect, test, and package. The Quickiematic needs 0.2 hours to manufacture, 0.35 hours to assemble, and 0.2 hours for the final set of operations. The VIP requires 0.7, 0.1, and 0.3 hours, respectively. In addition, there are 250 hours per week of manufacturing time available, 350 hours of assembly, and 150 hours total to inspect, test, and package.

Formulate this scheduling problem as a linear program that maximizes contribution.

1.3 Airline Optimization

This is problem #14 from Chapter 1 in *Applied Mathematical Programming*. An airline that flies to four different cities (A, B, C, D) from its Boston base owns 10 large jets (B707), 15 propeller-driven planes (Electra), and two small jets (DC9). The data in the table above (Figure 1) is available for the flight paths, assuming consistent flying conditions and passenger use.

(a) Can you choose decision variables that could be used in multiple linear programs based on this data set? What would a good selection of decision variables be, with no context on what the goal of the program would be?

(b) Write a constraint or set of constraints to ensure that City D is visited at least twice each day, and all the other cities are visited at least four times each day.

(c) Each plane can fly at most 18 hours in a single day. Write a constraint or set of constraints to govern the limitations on the availability of planes.

(d) Several objective functions could be pursued based on this information. Write objective functions designed to (i) minimize cost over a day, (ii) maximize profit over a day (note profit is revenue minus cost), and (iii) minimize fleet flying time over a day.

2 Solutions

2.1 Rabbit Food

(a) The decision variables are E and B, representing the quantity (in ounces) of each brand of food that you should be feeding your rabbit each day. The objective function will be in terms of food costs per day, and the constraints will be in terms of nutritional quantities per day.

$$\begin{aligned} \text{Minimize: } & z = 0.2E + 0.3B \\ \text{Subject to: } & E + B \leq 5 \quad \text{(Total food)} \\ & 12E + 8B \geq 24 \quad \text{(Fat)} \\ & E + 2B \geq 4 \quad \text{(Protein)} \\ & 12E + 12B \geq 36 \quad \text{(Carbs)} \\ & E, B \geq 0 \end{aligned}$$

(b) In standard form, we add non-negative slack variables for any less-than constraint, and we subtract non-negative slack variables for any greater-than constraint. This turns all constraints into equalities:

$$\begin{aligned} \text{Minimize: } & z = 0.2E + 0.3B \\ \text{Subject to: } & E + B + s_1 = 5 \quad \text{(Total food)} \\ & 12E + 8B - s_2 = 24 \quad \text{(Fat)} \\ & E + 2B - s_3 = 4 \quad \text{(Protein)} \\ & 12E + 12B - s_4 = 36 \quad \text{(Carbs)} \\ & E, B, s_1, s_2, s_3, s_4 \geq 0 \end{aligned}$$

In matrix form, we are maximizing the objective function $z = c^T x$ subject to the constraints $Ax = b$ and $x = 0$, where c, x, A, b are the following matrices and vectors:

$$c = \begin{pmatrix} 0.2 \\ 0.3 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, x = \begin{pmatrix} E \\ B \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix}, A = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 12 & 8 & 0 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 & -1 & 0 \\ 12 & 12 & 0 & 0 & 0 & -1 \end{pmatrix}, b = \begin{pmatrix} 5 \\ 24 \\ 4 \\ 36 \end{pmatrix}$$

(c) The figure on the next page has all 4 constraints labelled, and the feasible region outlined in black. The horizontal axis is for E-Z-Feed (E) and the vertical axis is for Bargain Bunny (B).

(d) This problem can be solved by identifying the corner points of the feasible region, plugging them into the objective function $z = 0.2E + 0.3B$, and seeing which one produces the smallest value. The feasible region has

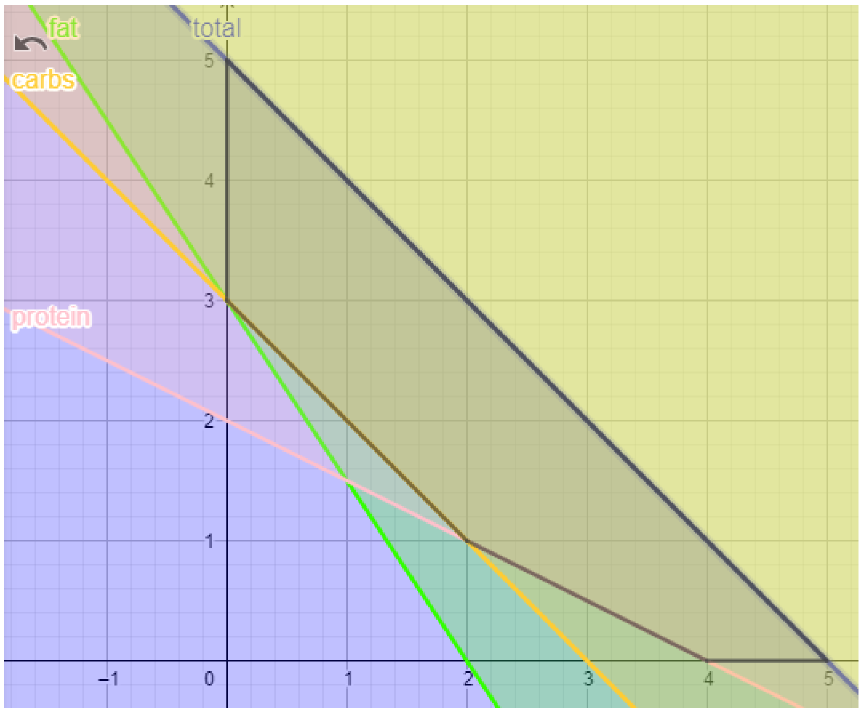

Figure 2: Rabbit Food Feasible Region

corners at $(0, 3), (0, 5), (5, 0), (4, 0), (2, 1)$; of these, the point $(2, 1)$ gives the smallest value in the objective function, yielding $z = 0.7$, or an expenditure of \$0.70 per day on rabbit food.

2.2 Camera Manufacturing

We are trying to maximize "contribution," which here means "profit" - thus the objective function will be in terms of the amount each camera model (Cub, C; Quickiematic, Q; and VIP, V) will be sold for. The decision variables will be the numbers of each type of camera to make each week. We need a constraint for each type of camera, ensuring that the minimum required quantity of each model will be made per week We also need a constraint for each production step, to guarantee that the time allocated for each step every week does not exceed the time available. Altogether, the program looks like this:

$$\begin{aligned}
\text{Maximize: } & z = 3C + 9Q + 25V \\
\text{Subject to: } & C \geq 250 \quad \text{(Cub minimum)} \\
& Q \geq 375 \quad \text{(Quickiematic minimum)} \\
& V \geq 150 \quad \text{(VIP minimum)} \\
& 0.1C + 0.2Q + 0.7V \leq 250 \quad \text{(Manufacturing time)} \\
& 0.2C + 0.35Q + 0.1V \leq 350 \quad \text{(Assembly time)} \\
& 0.1C + 0.2Q + 0.3V \leq 150 \quad \text{(Final operations time)}
\end{aligned}$$

Note that we have not included a non-negativity constraint for our decision variables. This is because the first three constraints already explicitly guarantee non-negativity, so there is no need to add the extra constraint. We can put this program in standard form by subtracting a non-negative slack variable from each of the first three constraints and adding a non-negative slack variable to each of the last three constraints, as well as specifically including a non-negativity constraint for these variables:

$$\begin{aligned}
\text{Maximize: } & z = 3C + 9Q + 25V \\
\text{Subject to: } & C - s_1 = 250 \quad \text{(Cub minimum)} \\
& Q - s_2 = 375 \quad \text{(Quickiematic minimum)} \\
& V - s_3 = 150 \quad \text{(VIP minimum)} \\
& 0.1C + 0.2Q + 0.7V + s_4 = 250 \quad \text{(Manufacturing time)} \\
& 0.2C + 0.35Q + 0.1V + s_5 = 350 \quad \text{(Assembly time)} \\
& 0.1C + 0.2Q + 0.3V + s_6 = 150 \quad \text{(Final operations time)} \\
& s_1, s_2, s_3, s_4, s_5, s_6 \geq 0
\end{aligned}$$

2.3 Airline Optimization

(a) We have information on how many of each plane is available and what the cost, revenue, and time is for each type of plane to fly each route. One immediate idea would be to create decision variables for each type of plane and each route, representing how many times per day that route is flown by that type of plane. However, this may not be fine-grained enough. You can always consolidate variables later if that suits a particular problem best! So we will make one decision variable for each **individual plane** for each route. For example, there are 10 large jets. We create variables $L_{A1}, L_{A2}, ..., L_{A10}$ to represent the number of trips taken to city A in a day by each of these 10 jets. Then we do the same thing for cities B, C, and D. We carry out this process for the propeller planes and small jets as well, for a total of $4 \cdot 10 + 4 \cdot 15 + 4 \cdot 2 = 108$ decision variables.

(b) For each city, we must add up all the decision variables associated with that city. So for city D, we would take $L_{D1} + L_{D2} + ...$ over all decision variables for city D, and end it by forcing the sum to be greater than or equal to 2. For the other cities, the same process occurs, and the only difference is that the sum must be greater than or equal to 4.

(c) Creating this constraint justifies our use of the very large number of decision variables. Each individual plane can only fly 18 hours, so each individual plane in the fleet now has a constraint based on the amount of time it takes to fly round-trip to each city. For example large jet #1 now has the constraint $L_{A1} + 2L_{B1} + 5L_{C1} + 10L_{D1} \leq 18$, where the coefficients are given by average flying time for the large jets to each city. This format persists to create the other constraints. This will yield a total of $10 + 15 + 2 = 27$ constraints. Note that we cannot construct these constraints just by having decision variables for each type of plane or for the fleet as a whole, as these would allow for a single plane to fly for over 18 hours if another plane flew for under 18 hours. Moreover, even if this explicit constraint was omitted, logically no single plane can fly for more than 24 hours in a day (assuming a standardized timekeeping structure), meaning that the use of the detailed decision variables is critical regardless.

(d) These objective functions will all be constructed in the usual way, with their coefficients taken from the appropriate columns of the table. To minimize cost, each decision variable is weighted with the cost of its corresponding plane type and destination; for example L_{A1} would have a coefficient of \$6,000, and so would L_{A2}, L_{A3}, and so on. Maximizing profit would be achieved by subtracting cost from revenue and using that as the coefficient, e.g. L_{A1} would have a coefficient of $\$5,000 - \$6,000 = -\$1,000$ for that objective function. Finally, minimizing overall fleet airtime would use the flight time for each plane type and destination as the coefficients, such that L_{A1} would have a coefficient of 1 whereas L_{B1} would have a coefficient of 2.