

APMA 1210 – HOMEWORK 4

Reading: Section 5.1 and the material on Game Theory

Problem 1: (8 points = 2 + 2 + 1 + 2 + 1 points) Consider the following LP

$$\begin{aligned} \max z &= 5x_1 + 3x_2 \\ \text{subject to } 5x_1 - 2x_2 &\geq 0 \\ x_1 + x_2 &\leq 7 \\ x_1 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

- (a) Find the optimal value and optimal vertex of the LP by hand. This means draw the feasible region (ok to do with a computer), find all the vertices (algebraically by hand), and calculating the values of z at the vertices (ok to use a calculator)
- (b) Find the shadow prices of the first and second constraints (by hand)
- (c) Write out the dual of the LP.
- (d) Using MATLAB, find the optimal value and optimal vertex of the dual LP. Take a screenshot of your code and the result
- (e) Check that your answers in (b) and (d) agree.

Date: Due: Wednesday, October 26, 2022 at 11:59 pm.

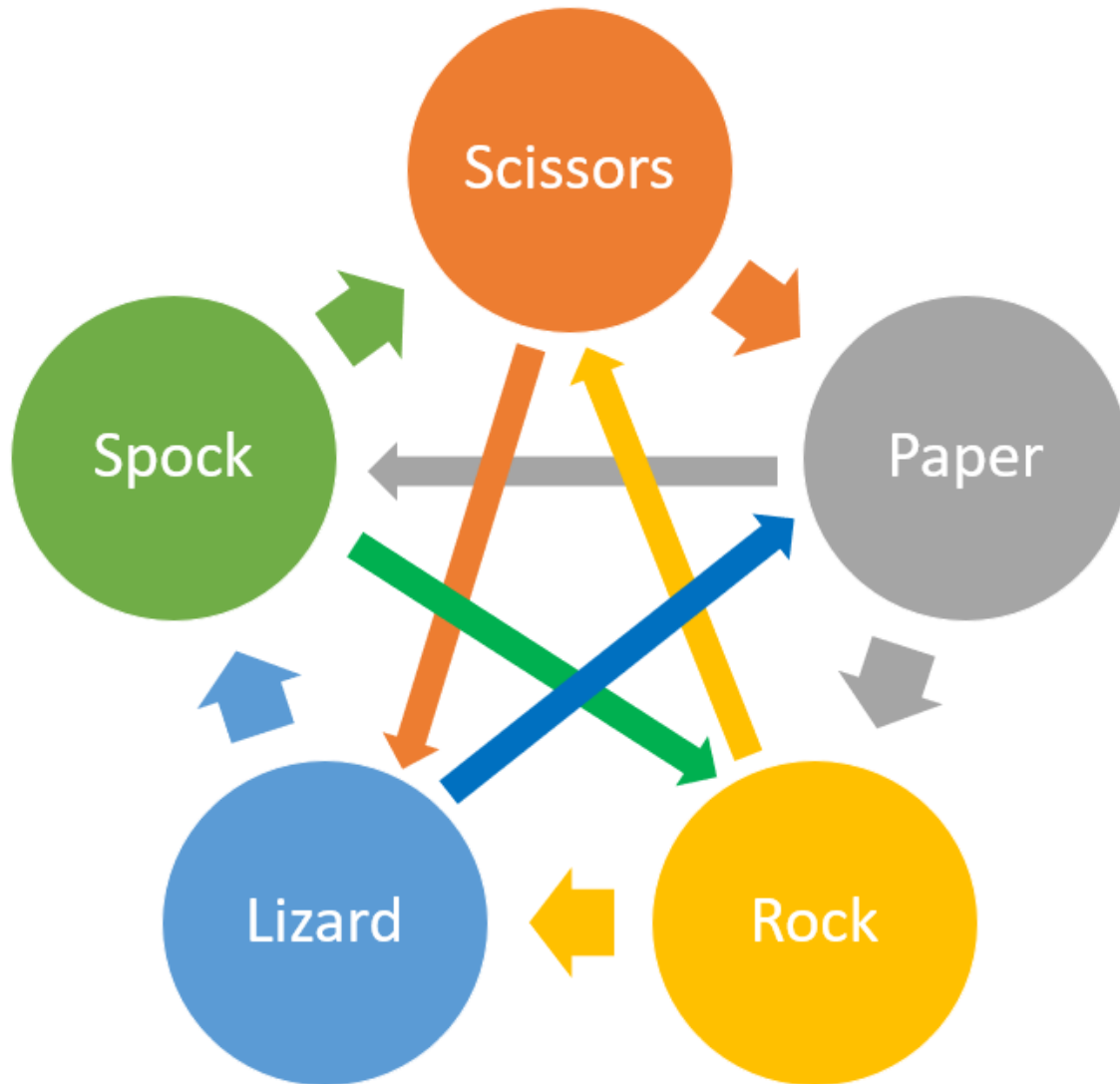
Problem 2: (5 points = 2 + 2 + 1 points) Solve the LP below as follows:

$$\begin{aligned} \max z &= -4x_2 + 3x_3 + 2x_4 - 8x_5 \\ \text{subject to } &3x_1 + x_2 + 2x_3 + x_4 = 3 \\ &x_1 - x_2 + x_4 - x_5 \geq 2 \\ &x_i \geq 0 \end{aligned}$$

- (a) Find the dual LP and solve it graphically, by hand. Ok to use computers to draw the graph, but you can't use MATLAB to solve the problem.
- (b) Calculate the slack variables e_j of the dual problem by hand and use complementary slackness to determine which variables x_j in the optimal primal solution are equal to zero.
- (c) Use the simplification in (b) to solve for the optimal solution and optimal vertex in the primal by hand (should be much easier now)

Note: If the ≥ 2 sign bothers you, feel free to write the second constraint as $-x_1 + x_2 - x_4 + x_5 \leq -2$

Problem 3: (7 points = 2 + 2 + 2 + 1 points) Let's play the following (Big Bang Theory-inspired) variation of Rock-Paper-Scissors called Rock-Paper-Scissors-Lizard-Spock (see picture on the next page). Assume two players are playing and that it's a zero-sum game. Winners get 2 dollars, losers lose 2 dollars, and tie gives 0 dollars.



(In case you have a hard time reading this: Scissors beat Paper and Lizard; Paper beats Rock and Spock; Rock beats Scissors and Lizard; Lizard beats Spock and Paper; Spock beats Scissors and Rock)

- (a) Write down the Gains Matrix G for Player 1. Please use the order (Scissors, Paper, Rock, Lizard, Spock)

- (b) Write down the LP problem for Player 1
- (c) Use MATLAB to find the optimal vertex and payoff for Player 1, as well as the optimal vertex of the dual problem (Player 2) using the `lambda` feature in `linprog`. Take a screenshot of your code and the result.
- (d) Are you surprised by the result in (c)? Try to explain why we got that answer: What would happen for example if Player 1 used scissors more than 20% of the time? Would that be a good strategy?