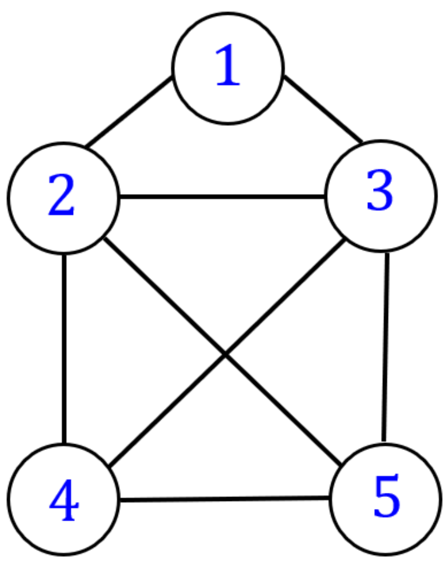
APMA 1210 – HOMEWORK 5

Reading: Sections 7.1 − 7.3

Problem 1: (3 = 2 + 1 points)

(a) Find the adjacency matrix of the following graph

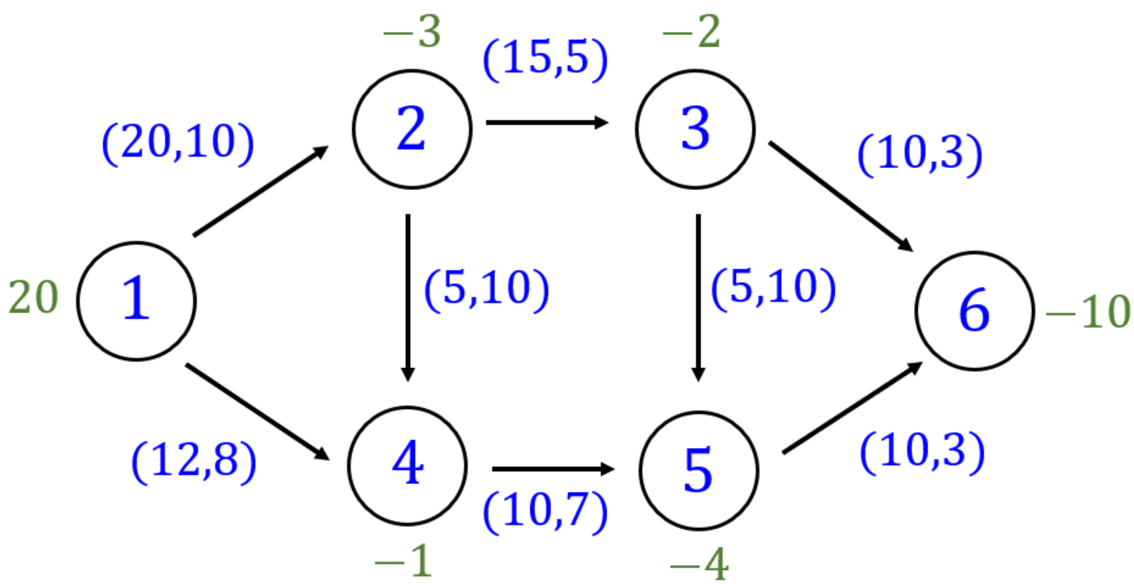
(b) Find the degree of each vertex



(Turn Page)

Date: Due: Wednesday, November 2, 2022 at 11:59 pm.

Problem 2: (10 = 2 + 2 + 5 + 1 points) Consider the following network



(a) Find the oriented incidence matrix M of the graph above (with lexicographic ordering of the edges)

(b) Set up the LP problem for the min flow (like the coffee transportation problem in lecture). Beware that here the intermediate vertices have their own demand as well!

(c) Use linprog in MATLAB to find the optimal vertex and optimal z-value for the problem in (b). Screenshot your code and results

(d) Redraw the graph from the problem, but label the edge (i, j) with the solution x_{ij} you found. For example if you found $x_{12} = 5$ then write "5" on the edge going from 1 to 2 (no need to put things like $(20, 10)$ or -3 on your graph)

Problem 3: (3 points) Suppose you have a general min flow problem

$$\min z = \sum_{i,j} c_{ij} x_{ij}$$
$$\text{subject to } \sum_{j} x_{ij} - \sum_{k} x_{ki} = b_i$$
$$l_{ij} \leq x_{ij} \leq u_{ij}$$

Find new variables y_{ij} that transforms the above problem into

$$\min z = \sum_{i,j} \overline{c_{ij}}\, y_{ij} + \sum_{i,j} \overline{d_{ij}}$$
$$\text{subject to } \sum_{j} y_{ij} - \sum_{k} y_{ki} = \overline{b_i}$$
$$0 \leq y_{ij} \leq \overline{u_{ij}}$$

Note: Please find the constants $\overline{c_{ij}}, \overline{d_{ij}}, \overline{b_i}, \overline{u_{ij}}$

Hint: How do you transform a constraint $a \leq b$ into $\star \geq 0$?

Problem 4: (4 points)

Show that any tree with n vertices has $n-1$ edges.

Hint: Induction on n. For the inductive hypothesis, remove a leaf of your tree; you may assume that the resulting object is still a tree.