APMA 1210 – MIDTERM 1 – STUDY GUIDE

The Midterm takes place on **Thursday, October 13, 2022** during the usual lecture time and in the regular lecture room. It will be an in-person exam and **NO** books/notes/calculators/cheat sheets will be allowed. The midterm counts for 15 % of your grade, and covers Lectures $1-9$ inclusive.

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. For a more thorough study experience, look at the lecture notes, homework, and practice exam. I will **NOT** ask you anything in the book that I haven't covered in lecture/homework/practice exam, so no need to read the book.

Format: There are *tentatively* $4-5$ questions on the exam, all of them free response, no multiple choice.

- One "put an LP problem in standard form" question
- One word problem where you have to give me the decision variables and objective function and constraints
- One simplex method question. I'll try to make the algebra reasonable
- One dual problem question
- Maybe an extra question

There will be **NO** coding on the exam, but there might be proofs

Date: Thursday, October 13, 2021.

Lecture 1: Intro to Operations Research

You can ignore this lecture, it's just a historical overview.

Lecture 2: Intro to Mathematical Programming

- Given a word problem, find the decision variables, objective function, constraints, and write down the complete optimization problem. Great practice questions are the examples from lecture, as well as the homework and recitation problems
- Write an LP problem in matrix form
- Also check out the Integer Linear Programming Problem from that lecture

Lecture 3: Linear Programming

- Put an LP problem in standard form. There are two rules: The Sign Rule and the Slack Variables Rule. I might ask you to write it in matrix form.
- Conceptually understand why the max/min has to be at a vertex, using the level curves (like $z = 10$ and $z = 15$ in the lecture example)
- Ignore the cookie problem

Lecture 4: Geometry of Feasible Regions and Convexity

- You don't need to memorize the definitions of polyhedron, hyperplane, and half-space, but just know what they are. For example, when I write $a^T x = b$, know that this represents a hyperplane.

- Remember that a is perpendicular to the hyperplane $a^T x = b$
- Know the definition of a line segment between x and y
- Know the definition of a convex set
- Show that a half-space $a^T x \leq b$ is convex (I would give you the definition of half-space)
- Know the definitions of convex combination and convex hull
- No need to know the alternative definition of a polyhedron
- You don't need to memorize the definitions 1 and 2 of a vertex, but please understand what they mean. For example, I would provide you the definition of an extreme point.
- Remember the proof with convexity on your homework.

Lecture 5: Extreme Points of Feasible Regions + Simplex Algorithm (I)

- Define active, basic solution, and basic feasible solution
- It's good to check out the examples that follow the definition, especially the pyramid one
- You can ignore the section on "Existence of Vertices"
- Ignore the proof in the section "Optimality and Extreme Points," but know the results, especially the "Glorious Result"
- The simplex method is done in detail in the following lecture

Lecture 6: Simplex Algorithm (II)

- I will definitely ask you an example of the simplex method, but I will probably just do something like 2 variables and 2 constraints, to make the algebra a bit more manageable
- Make sure to understand Example 1 in detail
- For the very last step, you don't need to write the constraints in terms of z_1 and z_2, since we know the point is already optimal. That said, you need to write the objective function in terms of z_1 and z_2
- Know how the higher-dimensional example presented in lecture or the homework works. If I ask a higher-dimensional example, it would be something like "Perform one step of the simplex method"
- Ignore the section "MATLAB Implementation." As a general rule, for the exams you can ignore the coding parts, that's more for the homework.

Lecture 7: Simplex Algorithm (III)

- Define local min (see homework), global min (see homework), and convex function
- Show that a local min of a convex function is a global min (see homework); I would give you the hints in this case
- Show that $z = c^T x$ is convex
- Understand the trick with $z_1 + \cdots + z_m$ to obtain a starting vertex. Here I'll make sure that the b_i are ≥ 0, to prevent any confusion

- Ignore the sections on "Degeneracy" and "Efficiency"

Lecture 8: LP Duality (I)

- Ignore the section on "Motivation" if you want, although I think that's pretty useful to understand why we have a min
- Define and find the dual problem to an LP problem
- You **don't** need to know all the weird sign rules
- Show that the dual of the dual is the primal
- Know the statements of the weak and strong duality theorems, but you don't need to know the proofs. Know how to prove the Corollary and the Co-Corollary

Lecture 9: LP Duality (II)

- You don't need to memorize the table with the possible scenarios, but know how to justify the ones with the X mark
- Know how to set up the LP problem for the farmer, and find the dual problem. No worries if you don't understand the "Interpretation part"
- Know how to set up the primal problem with slack and the dual problem with excess
- Know complementary slackness and the Example 2 that follows. For the proof, I would give you a hint like "Consider $y^T Ax$." No need to really understand the "Dual Problem" part that follows