APMA 1210 – MIDTERM 1

Name	
ID Number	
Signature	

Instructions: Welcome to your Midterm! You have 80 minutes to take this exam, for a total of 40 points. No books, notes, calculators, or cellphones are allowed. **Please write in complete sentences if you can.** Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, please clearly indicate so.

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Brown University Academic Code.

Date: Thursday, October 13, 2022.

1. *(8 points, 4 points each)*

   (a) Write the following LP problem in matrix form (the one with $c^T x$ and $Ax \leq b$). Do **NOT** solve it

$$\begin{aligned} \max\ & 2x_1 + 3x_2 + 5x_3 \\ \text{subject to } & x_1 - x_3 \leq 5 \\ & 2x_2 - 3x_3 \leq 4 \\ & x_1 + x_3 \leq 0 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

☐ Work on Scratch Paper

(b) Write the following LP in standard form (no need to use matrices here). Do **NOT** solve it

$$\begin{aligned} \max\ & x_1 + 3x_2 + 4x_3 \\ \text{subject to } & x_1 + 2x_2 \geq 6 \\ & x_2 + 2x_3 = 5 \\ & x_1 + x_2 - x_3 \leq 7 \\ & x_1, x_2 \geq 0 \\ & x_3 \text{ unconstrained} \end{aligned}$$

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2. *(4 points)* Write down the dual to the following LP, but do **NOT** solve it

$$\begin{aligned} \max\ & 2x_1 + 4x_2 + 8x_3 + 3x_4 \\ \text{subject to } & x_1 + 2x_2 + 2x_3 + x_4 \leq 3 \\ & x_2 + 3x_3 + 4x_4 \leq 6 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

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3. *(12 points)* Use the simplex method to solve the following LP, including the optimal vertex. At each step, clearly indicate which constraints are tight and which ones get released, and write down your change of variables. In the **last** step, no need to rewrite the constraints in terms of the new variables. You will not get any credit if you use another method.

$$\begin{aligned} \max\ & 2x_1 + 3x_2 \\ \text{subject to } & x_1 + 2x_2 \leq 4 \\ & 4x_1 + 2x_2 \leq 2 \\ & x_1 + 3x_2 \leq 9 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}$$

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4. *(8 points)* Sunrise Blends is a coffee company that produces n different types of coffee blends. Those types can be put into two categories: Regular Blends and Golden Blends.

   For each blend i, the company buys beans at s_i dollars/pound. Each blend requires r_i hours of roasting per pound, at t dollars/hour, and Sunrise Blends has a total of at most R daily roasting hours available

   In addition, the Golden Blends require some added gold, which costs g_i dollars per pound, and there is only at most G amount of gold available.

   The company sells each blend for q_i dollars per pound, and sells at least D pounds of coffee per day.

   Problem: Write a LP problem aiming to maximize profit (revenue minus cost) for Sunrise Blends while accounting for all reasonable restrictions. Clearly indicate your decision variables (including units), objective function, and constraints.

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5. *(8 points, 4 points each)*

   (a) Show using the **definition** of a convex set that the half-space $a^T x \leq b$ is convex (where a and b are vectors)

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(b) Show using the **definition** of a convex function that $f(x) = c^T x$ is convex (where c is a vector)

☐ Work on Scratch Paper

(Scratch Paper)