APMA 1210 Recitation 5

October 2022

1 Questions

1.1 Sensitivity Analysis

Consider a generic LP in matrix form:

$$\begin{aligned} \text{Maximize: } & z = c^T x \\ \text{Subject to: } & Ax \leq b \\ & x \geq 0 \end{aligned}$$

Suppose that this LP has a unique optimal value. Prove that its shadow prices are exactly the optimal vertex of its dual.

(Hint: consider a vector Δ that makes a *very small* change to the constraints of the primal LP, so that the constraints have the form $Ax \leq b + \Delta$. How does making this change affect the dual?)

1.2 Game Theory

Two companies, Alphabest Inc. and Bookster Ltd., are competing in the market of children's learning-to-read materials. Alphabest has opted to use customer-centric strategies to improve its market share: it gives out discount coupons (A1), offers home delivery services (A2), and includes free gifts with purchases (A3). Bookster prefers standard media advertising: its approaches are targeted internet ads (B1), newspaper inserts (B2), and magazine spreads (B3). The gains matrix is shown below:

	B1	B2	B3
A1	3	-4	2
A2	1	-7	-3
A3	-2	4	7

Write out the LPs for both companies to find their optimal strategies, where Alphabest has the goal of maximizing gains and Bookster has the goal of minimizing losses. Put them in a form that could be rapidly plugged into MATLAB. Identify the matrices and vectors that show that the problems are dual.

1.3 Intro to Networks

A gas company owns a pipeline network, sections of which are used to pump natural gas from its main field (S) to its distribution center (T). The company has two large pumping stations, A and B, by which the gas can reach the distribution center. At the present time, the company nets 1200 mcf (million cubic feet) of gas per month from its main field and must transport that entire amount to the distribution center. The following are the maximum usage rates and costs associated with each path the gas can take:

	S $\to$ A	S $\to$ B	A $\to$ B	A $\to$ T	B $\to$ A	B $\to$ T
Max use (mcf/month)	500	900	700	400	600	1000
Cost (dollars/month)	20	25	10	15	20	40

Draw this network. Include edge directions, label vertices with their supply/demand values, and label all edges with their capacities and costs. Construct the oriented incidence matrix M for this network. Write the LP associated with this problem, specifying what your decision variables mean and what units they have. Put your LP in matrix form and verify that M appears.