APMA 1210 – HOMEWORK 7

Reading: Sections 10.1 – 10.3 and 11.2 – 11.3

Problem 1: (5 points, Dynamic Programming)

A tech consultant starts out in Boston on Sunday and must be at a meeting in Providence on Thursday. For the three days in between (M,T,W) she can freelance in Boston, Providence or New Haven. She can earn \$120 a day in Providence, \$160 a day in Boston and \$170 a day in New Haven. Where should she spend the first three days and nights of the week to maximize her income less travel cost? Travel costs are shown below. Use Dynamic Programming to solve the problem.

From \ To	Providence (P)	Boston (B)	New Haven (N)
Providence (P)	0	50	20
Boston (B)	50	0	70
New Haven (N)	20	70	0

Problem 2: (5 points, Integer Programming)

Formulate the following as an Integer Program:

The Research and Development Division of Peyamazon has developed three possible new products. However, to avoid undue diversification of the company's product line, management has imposed the following

Date: Due: Wednesday, December 7, 2022 at 11:59 pm.

restrictions:

Restriction 1: From the three possible new products, exactly two should be chosen to be produced. Each of these products can be produced in either of two plants.

Restriction 2: Exactly one of the two plants should be chosen to be the sole producer of the new products. The production cost per unit of each product would be essentially the same in the two plants. However, because of differences in their production facilities, the number of hours of production time needed per unit of each product might differ between the two plants. These data are given in the Table below, along with other relevant information, including marketing estimates of the number of units of each product that could be sold per week if it is produced.

	Product 1	Product 2	Product 3	
Plant 1 Plant 2	3 hours 4 hours	4 hours 6 hours	2 hours 2 hours	30 hours 40 hours
Unit profit	5	7	3	(thousands of dollars)
Max units produced	7	5	9	(units per week)

Note: The second column is the production time used for each unit produced, and the third one is the production time available per week

The objective is to choose the products, the plant, and the production rates of the chosen products so as to maximize total profit.

Note: Feel free to use as many variables and constraints as you want.

Problem 3: (5 points, Non-Linear Programming)

Two companies are producing widgets. Suppose it costs the first company q_1 dollars to produce q_1 widgets and the second firm $\frac{1}{2}(q_2)^2$ dollars to produce q_2 widgets. If a total of q widgets are produced, consumers will pay $200 - q$ for each widget. If the two companies want to collude in an attempt to maximize the sum of their profit, how many widgets should each firm produce?

Set up the appropriate NLP problem and find the optimal solution using the techniques from lecture.

Problem 4: (5 points, 1 point each, Proof)

Suppose $f : \mathbb{R} \to \mathbb{R}$ is both concave and convex, that is, for all x and y and all $0 \leq \lambda \leq 1$ we have

$$f(\lambda x + (1-\lambda)y) = \lambda f(x) + (1-\lambda) f(y)$$

Show that $f(x) = ax + b$ for some constants a and b, using the steps below.

STEP 1: Show it is enough to assume $f(0) = 0$ and to show that $f(x) = ax$ for some a

Hint: Consider $g(x) = f(x) - f(0)$, show that $g(0) = 0$, g is both concave and convex, and that $g(x) = ax$ implies $f(x) = ax + b$ for some b

So from now on assume $f(0) = 0$ and our goal is to show $f(x) = ax$ for some a

STEP 2: Show that if $c \geq 0$ then $f(cx) = cf(x)$

Hint: Check the cases $c = 0$ and $c = 1$ separately. If $0 < c < 1$, notice that $cx = cx + (1-c)0$ and if $c > 1$, notice that $x = \frac{1}{c}(cx) + \left(1 - \frac{1}{c}\right)0$

STEP 3: Show that for all x and y we have $f(x+y) = f(x) + f(y)$

Hint: Use the formula with $\lambda = \frac{1}{2}$ and the previous step

STEP 4: Show that if $c < 0$, then $f(cx) = cf(x)$

Hint: First show this for $c = -1$ using $x + (-x) = 0$ and the previous step. Then, if $c < 0$, notice $c = -(-c)$

Now you've shown that $f(cx) = cf(x)$ for all c

STEP 5: Show that $f(x) = ax$ for some a.

Hint: Notice $x = x\,(1)$ and you can pull the x out of the $f(x)$