SOLUTIONS

PROBLEM 1

(a)

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b)

$$M = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccccccc} X_{12} & X_{13} & X_{23} & X_{24} & X_{25} & X_{35} & X_{36} & X_{45} & X_{56} \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

(c)

$$\text{MIN} \quad c^T x$$

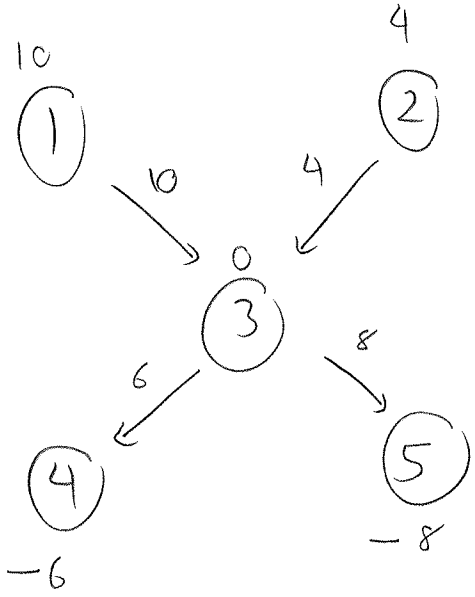
$$\text{SUBJECT TO} \quad Mx = b$$

$$\text{AND} \quad 0 \le x_{ij} \le 25$$

$$c = \begin{bmatrix} 4 \\ 5 \\ 1 \\ 0 \\ 4 \\ 4 \\ 3 \\ 2 \\ 4 \end{bmatrix} \qquad b = \begin{bmatrix} 20 \\ -5 \\ 10 \\ -5 \\ 0 \\ -20 \end{bmatrix}$$

PROBLEM 2 |

STEP 1 INITIAL TREE

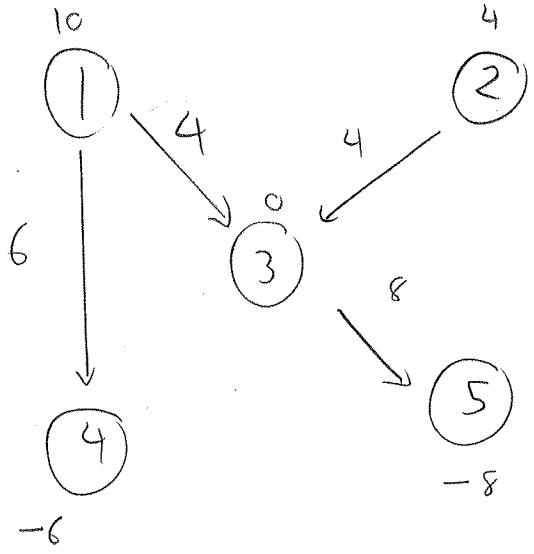


STEP 2 $\overline{C_{14}} = C_{14} - C_{34} - C_{13} = 1 - 1 - 8 = -8 < 0$

SO ADD EDGE $1 \rightarrow 4$

IN THAT CASE $X^* = 6$ AND $3 \rightarrow 4$ GETS ELIMINATED

SO WE GET

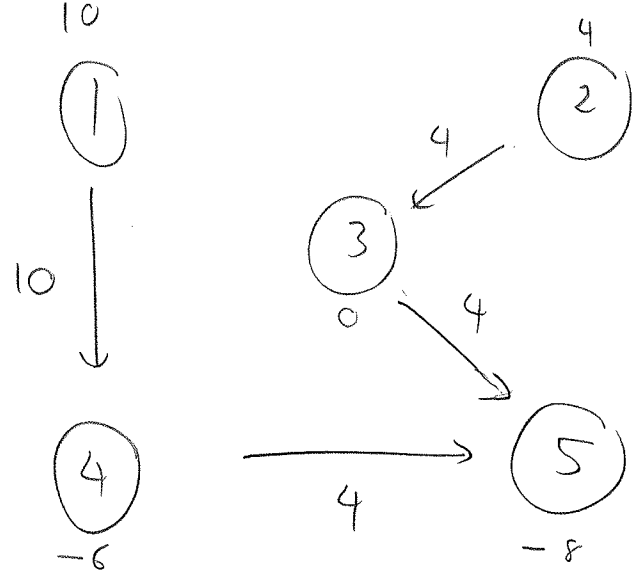


STEP 3 $\overline{C_{45}} = C_{45} - C_{35} - C_{13} + C_{14} = 10 - 4 - 8 + 1 = 11 - 12 = -1 < 0$

SO ADD EDGE $4 \rightarrow 5$

IN THAT CASE $X^* = 4$ AND $1 \rightarrow 3$ GETS REMOVED

SO WE GET



STEP 4 $\overline{C_{52}} = C_{52} + C_{23} + C_{35} = 7 + 2 + 4 = 13 > 0$

$\overline{C_{12}} = C_{12} + C_{23} + C_{35} - C_{45} - C_{14}$

$= 10 + 4 + 4 - 4 - 10 = 4 > 0$

$\overline{C_{13}} = C_{13} + C_{35} - C_{45} - C_{14} = 8 + 4 - 10 - 1 = 1 > 0$

$\overline{C_{34}} = C_{34} + C_{45} - C_{35} = 1 + 10 - 4 = 7 > 0$

ALL THE $\overline{C_{ij}}$ ARE > 0 SO THIS TREE IS OPTIMAL

OPTIMAL TREE SEE ABOVE

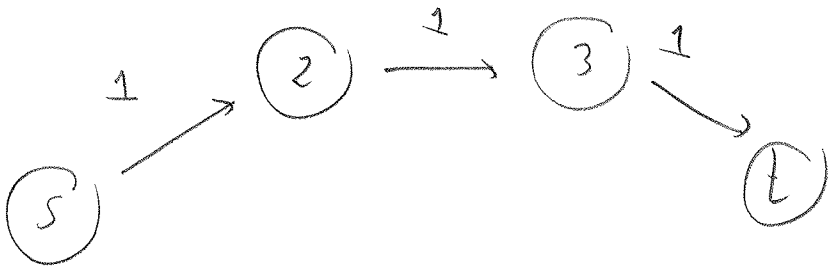
OPTIMAL COST $Z = 1 \times 10 + 4 \times 10 + 4 \times 4 + 4 \times 2$

$= 10 + 40 + 16 + 8$

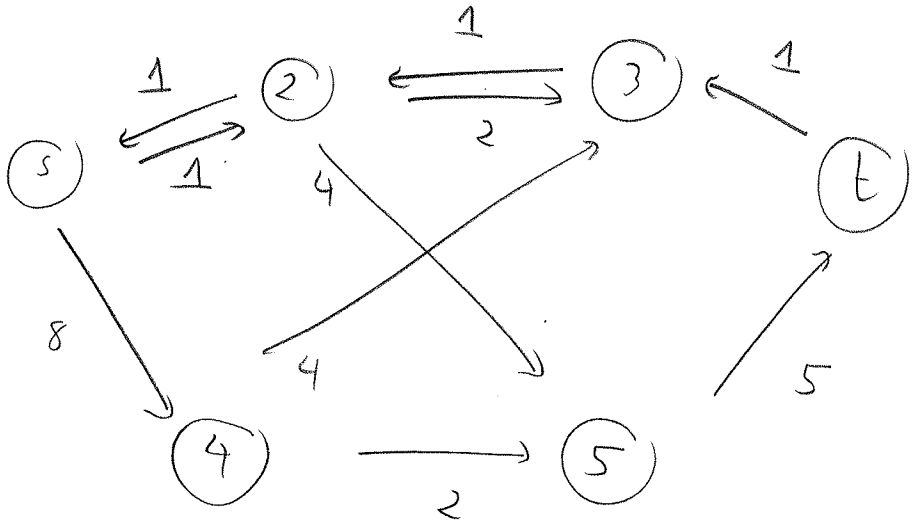
$= 74$

PROBLEM 3

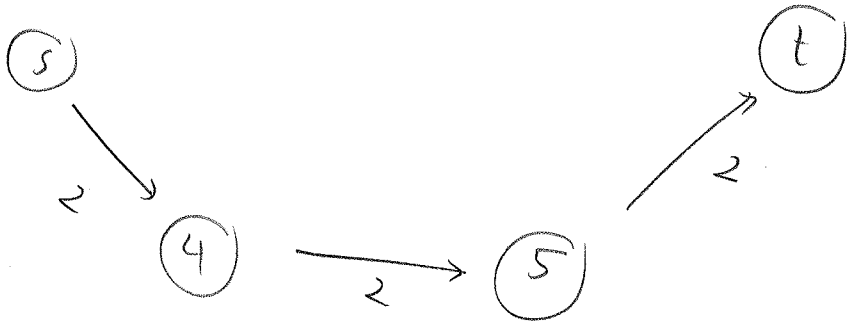
(a) STEP 1 PATH



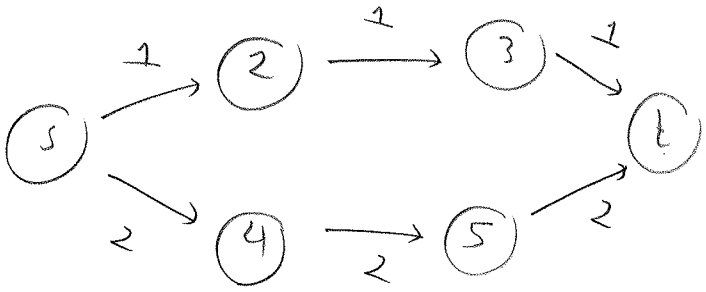
STEP 2 RESIDUAL GRAPH



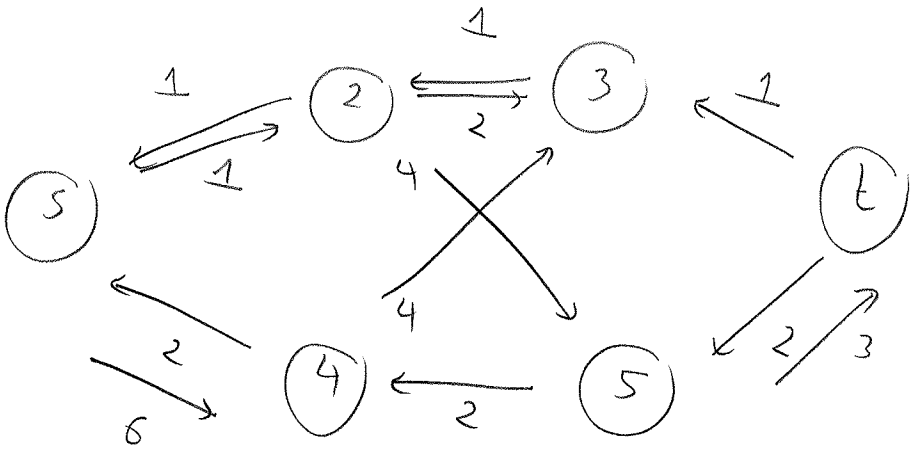
STEP 3 PATH (IN RESIDUAL)



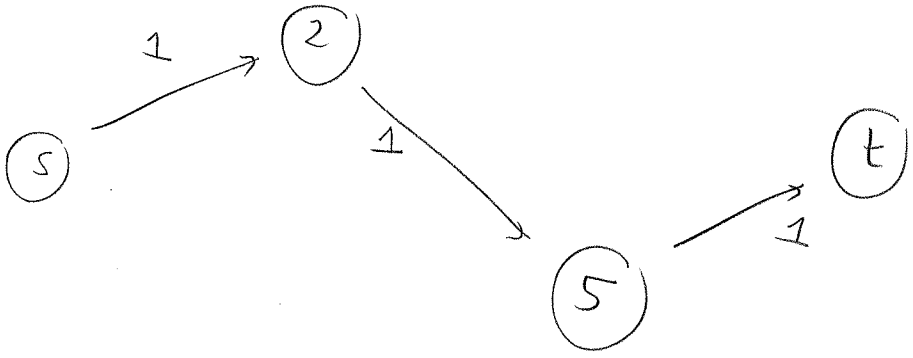
STEP 4 AUGMENTED GRAPH (STEP 1 + STEP 3)



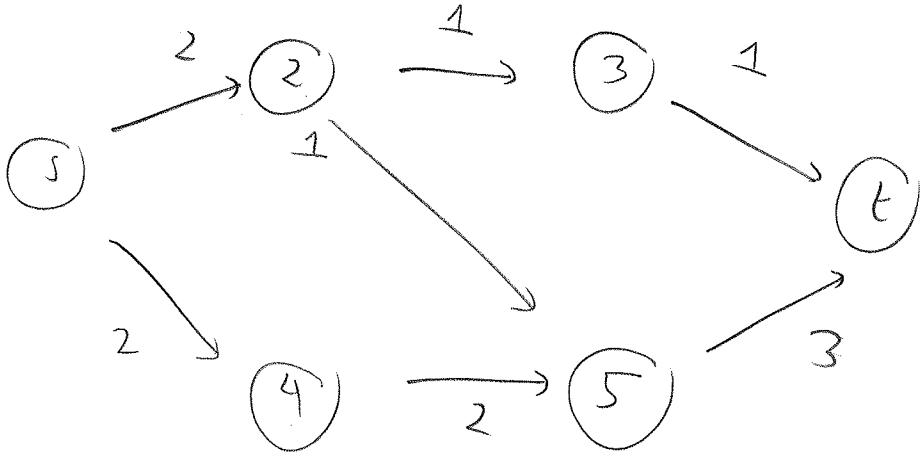
STEP 5 RESIDUAL (OF STEP 4)



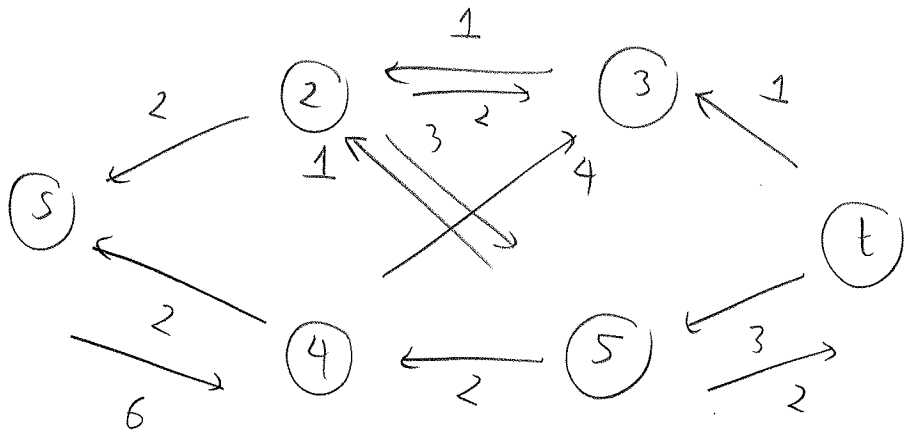
STEP 6 PATH (IN RESIDUAL)



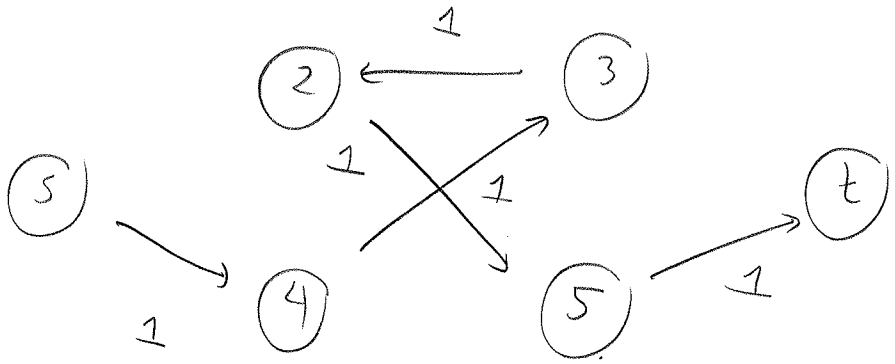
STEP 7 AUGMENTED GRAPH (STEP 4 + STEP 6)



STEP 8 RESIDUAL (OF STEP 6)

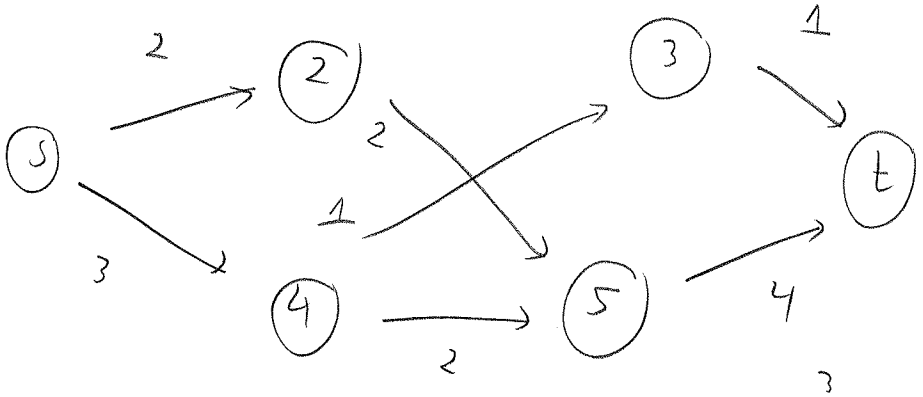


STEP 9 PATH ! (IN RESIDUAL)

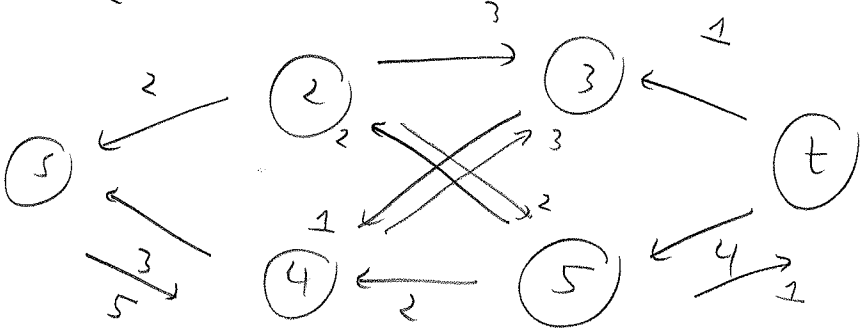


WOW!

STEP 10 AUGMENTED GRAPH (STEP 7 + STEP 9)



STEP 11 RESIDUAL

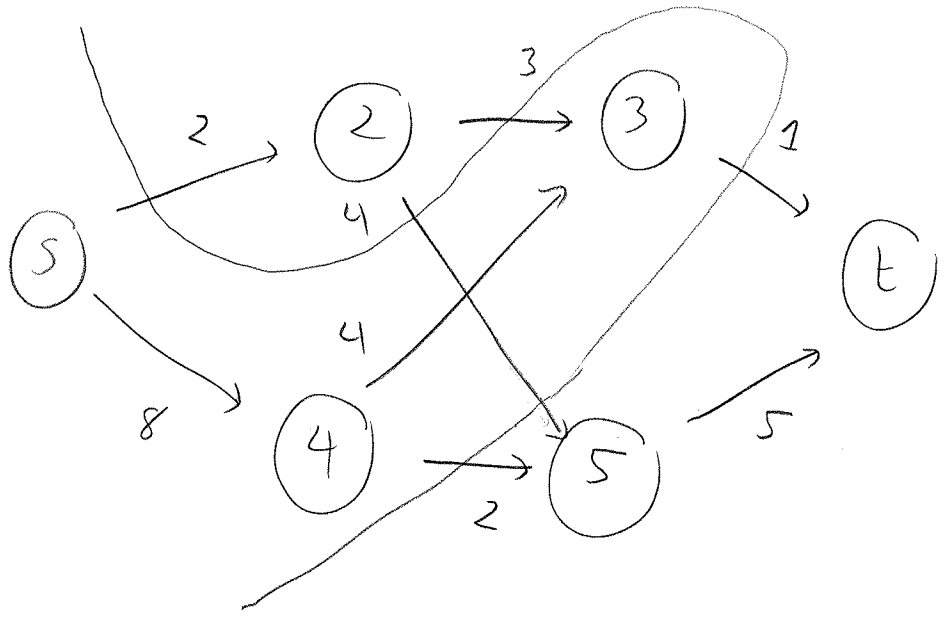


STOP

ANSWER OPTIMAL EDGES: SEE (STEP 10)

OPTIMAL VALUE: $2+3=5$

(b)



THE CUT $\{s, 3, 4\}$ WORKS

THE EDGES POINTING FROM THE $\{s, 3, 4\}$ TO THE $\{2, 5, t\}$ REGION ARE

$s \to 2$ (VALUE 2)

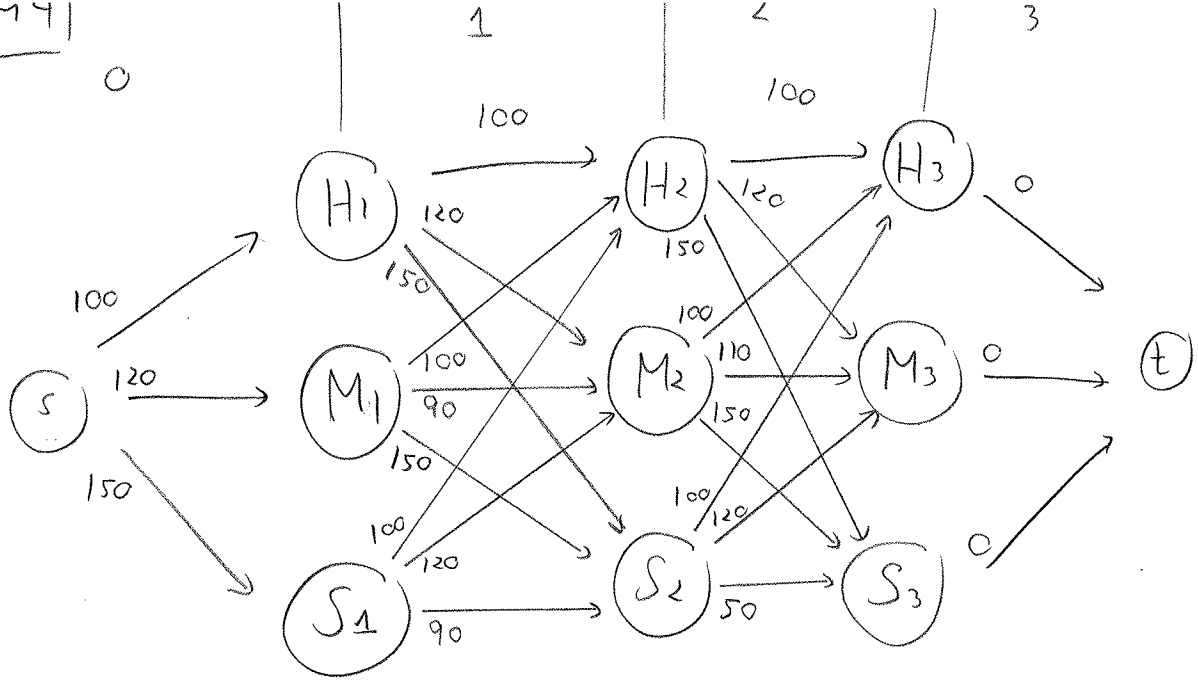
$3 \to t$ (VALUE 1)

$4 \to 5$ (VALUE 2)

SO THE VALUE OF THE CUT IS $2+1+2=5$

NOTE $2 \to 5$ POINTS FROM $\{2, 5, t\}$ TO $\{2, 5, t\}$ SO WE DON'T INCLUDE IT!

PROBLEM 4



STAGE 3

DEP \ ARR	t	SMALLEST COST	TRIP
H_3	0	0	$H_3 \rightarrow t$
M_3	0	0	$M_3 \rightarrow t$
S_3	0	0	$S_3 \rightarrow t$

STAGE 2

D \ A	$H_3 \rightarrow t$	$M_3 \rightarrow t$	$S_3 \rightarrow t$	SMALL	TRIP
H_2	(100)	120	150	100	$H_2 \rightarrow H_3$
M_2	(100)	110	150	100	$M_2 \rightarrow H_3$
S_2	100	120	(50)	50	$S_2 \rightarrow S_3$

STAGE 1

D \ A	$H_2 \rightarrow t$	$M_2 \rightarrow t$	$S_2 \rightarrow t$	SMALL	TRIP
H_1	$100+100=(200)$	$120+100=220$	$150+50=(200)$	200	$H_1 \rightarrow H_2$ $H_1 \rightarrow S_2$
M_1	$100+100=(200)$	$90+100=190$	$150+50=(200)$	200	$M_1 \rightarrow H_2$ $M_1 \rightarrow S_2$
S_1	$100+100=200$	$120+100=220$	$90+50=(140)$	140	$S_1 \rightarrow S_2$

STAGE 0

D \ A	$H_1 \rightarrow t$	$M_1 \rightarrow t$	$S_1 \rightarrow t$	SMALL	TRIP
S	$100+200=300$	$120+200=220$	$150+140=(290)$	290	$S \rightarrow S_1$

BEST TRIP $S \rightarrow S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow t$

BEST VALUE # 290