

## APMA 1210 – MIDTERM 2 – STUDY GUIDE

The Midterm takes place on **Tuesday, November 15, 2022** during the usual lecture time and in the regular lecture room. It will be an in-person exam and **NO** books/notes/calculators/cheat sheets will be allowed. The midterm counts for 20% of your grade, and covers Lectures 10 – 17 inclusive. It won't be cumulative, but remember the material itself is cumulative, so you would need to know for example what a Dual LP problem is and how to set it up. On the other hand, I will **not** ask you things like the simplex algorithm from midterm 1 where you change coordinates.

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. For a more thorough study experience, look at the lecture notes, homework, and practice exam. I will **NOT** ask you anything in the book that I haven't covered in lecture/homework/practice exam, so no need to read the book.

**Format:** There are *tentatively* 5–6 questions on the exam, all of them free response, no multiple choice.

- Find the shadow price of one (or several) constraints, both using the definition and the Dual LP
- Set up a game theory problem using LP
- Find the adjacency and oriented incidence matrix of a graph, and write a network flow problem in terms of LP

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*Date:* Tuesday, November 15, 2022.

- Implement the Network Simplex Algorithm, including finding the initial tree and checking if a tree is optimal
- Solve a Max flow problem, using residual graphs and min cut
- Solve a problem using dynamic programming

There will be **NO** coding on the exam, but there might be word problems and/or proofs

### LECTURE 10: SENSITIVITY ANALYSIS

- Define shadow price
- Thoroughly study the example from lecture, my guess is that the midterm problem would be something similar
- Know that there's a relation between shadow prices and dual LP problems
- (The example used slack variables, but you don't need to remember those)
- Define reduced cost and understand the relation with dual LP problems
- Ignore the section "Other Sensitivity Issues"
- Homework 4: Problems 1 and 2, although remember that there's no MATLAB allowed

## LECTURE 12: GAME THEORY

- Find the gains matrix of a game, like here Rock/Paper/Scissors
- Understand what a mixed strategy is
- Define expected payoff (for player 1)
- *Thoroughly* understand the Pokémon example and how it leads to a LP problem
- Understand what this has to do with Dual LP problems
- Ignore the section on MATLAB implementation
- Homework 4: Problem 3

## LECTURE 13 + 14: NETWORK PROBLEMS

- Define: Graph, Directed Graph
- You don't need to understand the Google Graph
- Find the Adjacency Matrix and the Oriented Incidence Matrix of a graph
- *Thoroughly* the coffee problem and how it leads to an LP problem, as well as all the vocabulary associated to it like capacity constraints and conservation of flow; this is continued in the next lecture
- Understand all the variations talked about in lecture: Transportation Problem, Assignment Problem, Max Flow, Shortest Path. I could ask you a word problem with those

- Define: Connected, Disconnected, Cycle, Tree, Degree, Leaf, Branch
- Homework 5: Problems 1, 2, 3, 4

### LECTURE 15: NETWORK SIMPLEX ALGORITHM

- Show that every finite tree has a leaf
- Show that a tree with  $n$  vertices has  $n - 1$  edges
- Know the two other facts about trees, but you don't need to prove them
- Understand the correspondence between Spanning Trees and Invertible  $3 \times 3$  sub-matrices in the example in lecture, it is very interesting
- Find a tree solution (initial vertex) of a graph, you need this to start the network simplex algorithm
- You don't need to know the technical definition of a tree solution
- Implement the network simplex algorithm to find an optimal tree; this is continued in the next lecture. Notice this involves:
  - (1) Finding an initial tree, using the algorithm above
  - (2) Checking if the initial tree is optimal, using the reduced cost
  - (3) If not, adding/removing an edge
  - (4) Checking if the new tree is optimal, using the reduced cost
- Homework 6: Problem 1

## LECTURE 16: MAX FLOW/MIN CUT

- Solve a max flow problem using the method of residual graphs
- Define cut
- Notice a cut doesn't have to be a line, and for the value of the cut you only consider edges pointing out of the  $s$ -region
- Solve a max flow problem using min cuts
- Understand that max flow = min cuts
- Show that a graph with  $n$  intermediate cities has  $2^n$  cuts.
- Homework 6: Problems 2, 3, 4

## LECTURE 17: DYNAMIC PROGRAMMING

- Understand how dynamic programming works, and in particular how you use your friend  $f$  to solve the problem implicitly.
- You can ignore the section “Ordering on vertices,” in the problem that I would give the ordering will be clear
- Solve a problem using dynamic programming, you would need to draw all the tables and give me the shortest path and total cost
- Ignore the section “Distance between Words”