APMA 1210 – MIDTERM 2

Name	
Banner ID	
Signature	

Instructions: Welcome to your Midterm! You have 80 minutes to take this exam, for a total of 40 points. No books, notes, calculators, or cellphones are allowed. **Please write in complete sentences if you can.** Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on scratch paper, please check the box "Work on scratch paper"

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Brown University Academic Code.

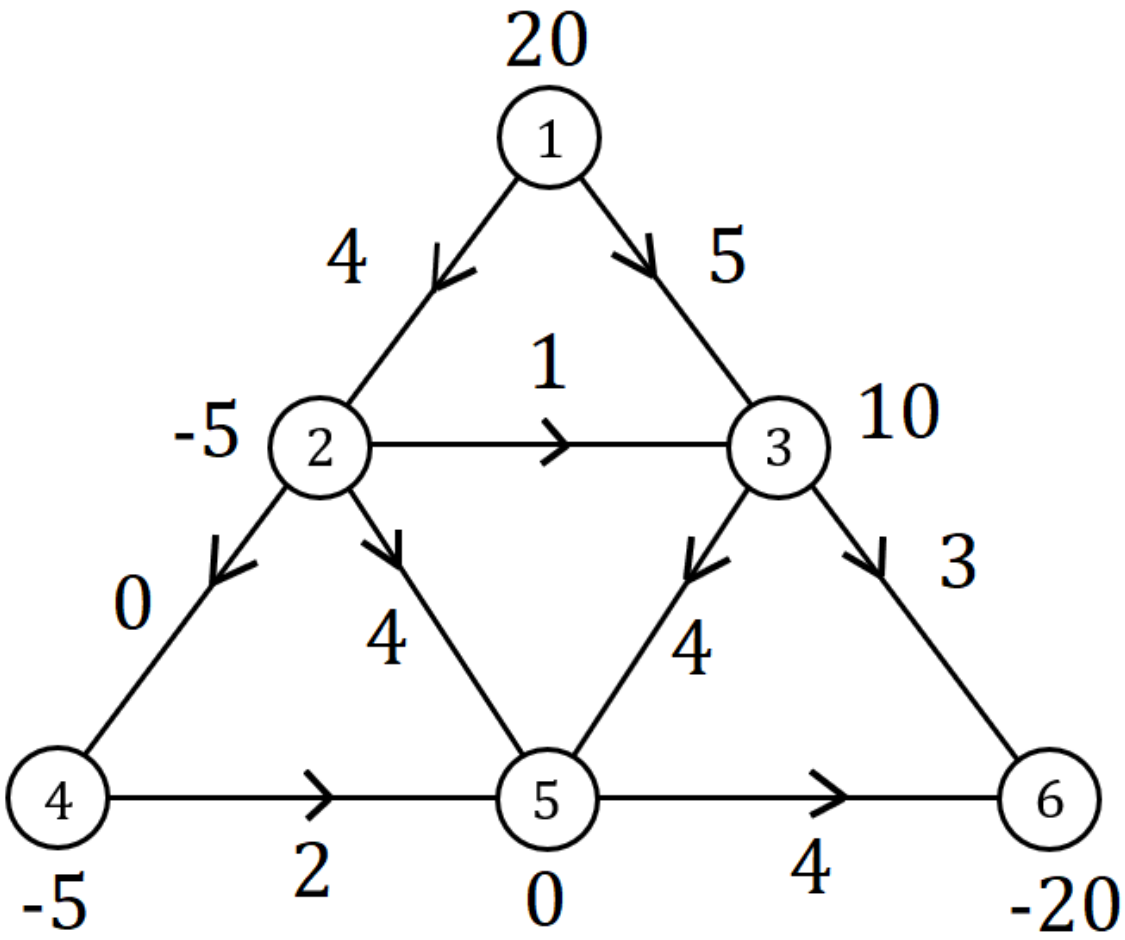
Date: Tuesday, November 15, 2022.

1. *(8 = 2 + 3 + 3 points)* In the graph below, with lexicographic ordering

   (a) Find the adjacency matrix A

   (b) Find the oriented incidence matrix M

   (c) Set up, but do **NOT** solve, the network flow LP problem that minimizes the total cost. The number on each edge is the **COST**. Assume each max capacity constraint is 25. Ok to use matrices here.

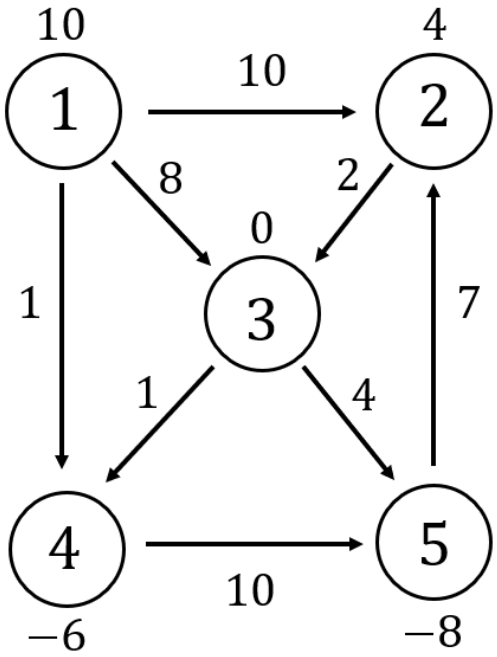


(Space to do Problem 1)

☐ Work on Scratch Paper

2. *(10 points)* Use the network simplex algorithm to find the optimal tree and optimal (minimal) total cost z of the following network. The numbers on the edges are the **COSTS** on each edge. Show all your work, and don't forget to calculate the reduced cost at each stage.

   Note: Start with the tree $1 \to 3$, $2 \to 3$, $3 \to 4$, $3 \to 5$



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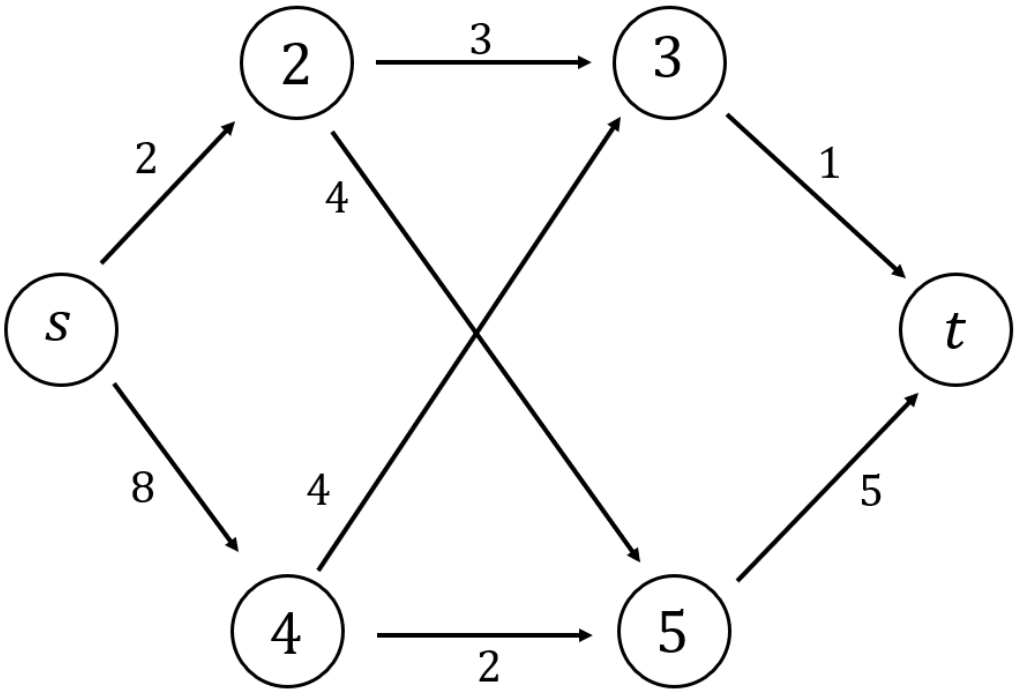
(Space to do Problem 2)

3. *(12 = 9 + 3 points)*

   (a) Solve the max-flow problem (optimal edges and optimal value) for the following network using the method of residual graphs. Show all your steps

   (b) On the graph below, draw a cut whose value is equal to the max-flow value you found.

Note: The answer is not 4. Don't ignore the reversed edges!



(Space to do Problem 3)

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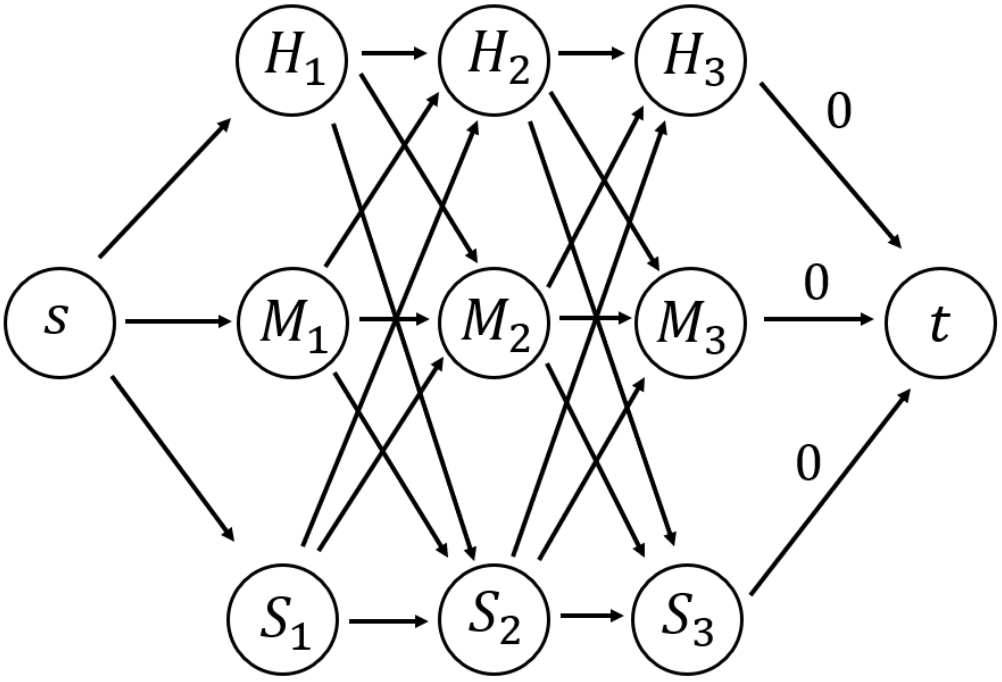
4. *(10 points)* Dr. Peyam is going to a conference in Boston and needs to stay at hotels for a total of 3 nights (Monday, Tuesday, and Wednesday nights). The rates are below, and on each night, Dr. P can choose which hotel he wants to stay at. The rate for each night needs to be paid upfront.

   - **Hill-ton:** \$100 per night
   - **Meow-rriott:** \$120 per night. If you stay there on a Monday night, then your Tuesday night rate is \$90. If you stay on a Tuesday night, then your Wednesday night rate is \$110.
   - **Share-a-ton:** \$150 per night. If you stay on a Monday night, then your Tuesday night rate is \$90. If you stay on a Tuesday night, your Wednesday night rate is \$50

   This gives the following network



   Help Dr. P out! Use dynamic programming to find the optimal cost and the optimal path(s) of his 3 night stay. Show all your work, **including the tables.**

(Space to do Problem 4)

☐ Work on Scratch Paper

(Scratch Paper)