APMA 1210 Midterm 2 Practice Exam

November 2022

1 Shadow Prices

Recall the rabbit food optimization problem from an early recitation, where the goal was to find a way to meet a rabbit's nutritional needs on a tight budget. The LP for calculating the optimal mix of rabbit food to minimize cost was:

$$\begin{aligned} \text{Minimize: } & z = 0.2E + 0.3B \\ \text{Subject to: } & E + B \leq 5 \quad \text{(Total food)} \\ & 12E + 8B \geq 24 \quad \text{(Fat)} \\ & E + 2B \geq 4 \quad \text{(Protein)} \\ & 12E + 12B \geq 36 \quad \text{(Carbs)} \\ & E, B \geq 0 \end{aligned}$$

The optimal solution is $E = 2, B = 1$ with $z = 0.7$ as the optimal value.

(a) Find the shadow prices for all four constraints through **reasoning and/or algebraic computation** (without using the dual problem).

(b) Find the dual problem and solve it to compute the shadow prices. Make sure they are the same as in part (a).

2 Game Theory

Two people are playing a game. In each round, both players simultaneously hold up either 1 finger or 2 fingers. Let n be the total number of fingers being held up in a particular round. If n is odd, Player 1 pays Player 2 n dollars; if n is even, Player 2 pays Player 1 n dollars.

(a) Construct the gains matrix for this game, with Player 1's decisions as the rows and Player 2's decisions as the columns.

(b) Set up the LPs for finding the optimal strategy for each player (no need to solve).

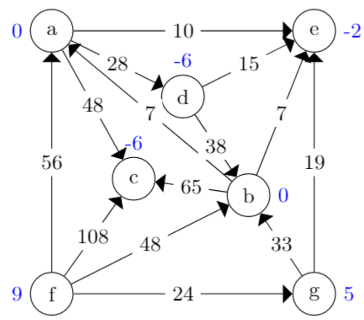

Figure 1: Network for Problem 4

3 Setting Up a Network LP

You are travelling to a remote camping location in the mountains, where roads can be very long and winding and access is very limited. There are 5 towns between you and your campsite, but not all of them have roads to each other, and not all of them have roads to the campsite. The following table shows the road distance in miles for the direct roads between each location, travelling from the town corresponding to the row to the town corresponding to the column. A dash indicates that no direct road exists between the two locations.

	A	B	C	D	E	Campsite
Origin	40	60	50	-	-	-
A		10	-	70	-	-
B			20	55	40	-
C				-	50	-
D					10	60
E						80

(a) Draw the network associated with this problem, including edge directions, and label it with costs and capacities as appropriate. Construct the oriented incidence matrix for the network, making sure it is clear which rows and columns correspond to which vertices and edges.

(b) Write an LP for finding the shortest path from your starting point to your campsite (do not solve).

4 Network Simplex Algorithm

Consider the network shown at the top of this page. It has 7 vertices, each labelled with their demands in blue, and a variety of edges with their costs in black. Apply the network simplex algorithm to this network, starting from the spanning tree formed by the set of edges $\{(a,d),(b,a),(f,a),(f,c),(g,b),(g,e)\}$.

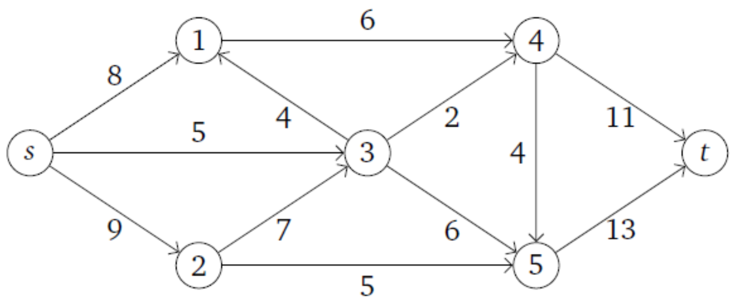

Figure 2: Network for Problem 5

5 Max Flow and Min Cut

Consider the network given at the top of this page, for which the capacities of the edges are labelled. Use the residual method to find the maximum flow through this network. Draw a curve that gives the minimum cut for the network.

6 Dynamic Programming

A construction company has three projects in progress. According to the current allocation of manpower, equipment, and materials, the three projects can be completed in 15, 20, and 18 weeks. Management wants to reduce the completion times and has decided to allocate an additional $20,000 across all three projects. The new completion times as functions of the additional funds allocated to each projects are given in the table below. How should the $20,000 be allocated among the projects to achieve the largest total reduction in completion times? Assume that the additional funds can be allocated only in blocks of $5,000. (*Hint: think of each stage as being the amount of money left to be assigned.*)

Extra funds (in $1,000s)	Project 1	Project 2	Project 3
0	15	20	18
5	12	16	15
10	10	13	12
15	8	11	10
20	7	9	9