APMA 1210 Practice Final

December 2022

1 Simplex method

Use the simplex method to solve the following LP:

$$\begin{aligned}
\text{Maximize: } & z = 3x_1 + x_2 + 3x_3 \\
\text{Subject to: } & 2x_1 + x_2 + x_3 \leq 2 \\
& x_1 + 2x_2 + 3x_3 \leq 5 \\
& 2x_1 + 2x_2 + x_3 \leq 6 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

2 Dynamic programming

The following table specifies the unit weights and values of five products held in storage. The quantity of each item is unlimited.

Product	Weight W_i	Value V_i
1	7	9
2	5	4
3	4	3
4	3	2
5	1	$\frac{1}{2}$

A plane with a capacity of 13 weight units is to be used to transport the products. How should the plane be loaded to maximize the value of goods shipped? (Formulate the problem as an integer program and solve by dynamic programming.)

3 Max flow/min cut

Use the residual graphs method to find the maximum flow on the network below, where the edges are labelled with their capacities. Based on this information, identify a minimum-weight cut for the network.

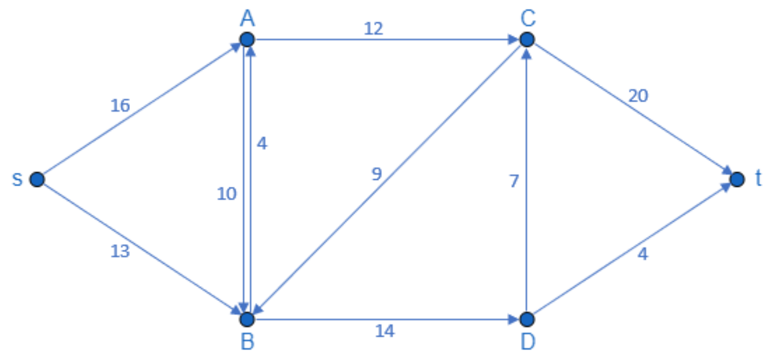

Figure 1: Network for Max Flow question

4 Network simplex method

In the network below, the edges drawn in orange form a spanning tree of the graph. All edges are labelled with their costs, with blue corresponding to edges outside the initial spanning tree and orange corresponding to edges belonging to the initial spanning tree. Starting from this tree, use the network simplex algorithm to find the minimum cost spanning tree.

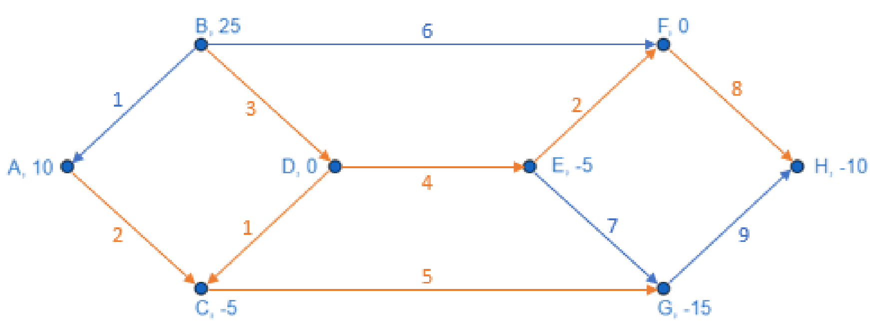

Figure 2: Network for Simplex question

5 Integer programming

Graph the following integer program:

$$\begin{aligned} \text{Maximize: } & z = x_1 + 5x_2 \\ \text{Subject to: } & -4x_1 + 3x_2 \leq 6 \\ & 3x_1 + 2x_2 \leq 18 \\ & x_1, x_2 \geq 0,\ x_1, x_2 \in \mathbb{Z} \end{aligned}$$

Apply the branch-and-bound procedure, graphically solving each linear-programming problem encountered. Interpret the branch-and-bound procedure graphically as well.

6 Nonlinear programming

A company produces two kinds of products A and B. Both of them are produced in two steps. In the first step, the raw materials are made into prototype C, and in the second step, the prototypes are further processed into A and B. For the first step, the cost of making prototype C is $\frac{1}{2}x_C^2$, where x_C is the number of prototype C. For the second step, prototype C is processed into the same number of either A or B. The costs for further processing in the second step are

$$\begin{aligned} & x_A \quad \text{where } x_A \text{ is the number of product A} \\ & \frac{1}{2}x_B^2 \quad \text{where } x_B \text{ is the number of product B} \end{aligned}$$

And the revenues for selling each kind of product are

$$\begin{aligned} & 10x_A \quad \text{where } x_A \text{ is the number of product A} \\ & 16x_B \quad \text{where } x_B \text{ is the number of product B} \end{aligned}$$

Now the company has a limited raw material supply, so it can produce at most 20 units of prototype C. Try to maximize the profit of the company by finding how many products of A and B should be produced.

7 Convexity

Suppose that you have two functions $f(x)$ and $g(x)$ that are both convex. Prove that the function $h(x) = \max\{f(x), g(x)\}$ is also convex.