MATH 308 – FINAL EXAM

Name	
Student ID	
Section	509
Signature	

Instructions: Welcome to your Final Exam! You have 120 minutes to take this exam, for a total of 70 points. No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your final answer, but also on your work. **Please put your answers in the boxes provided.** If you need to continue your work on a scratch paper, please check the box “Work on Scratch Paper,” or else your work will be discarded.

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Aggie Honor Code.

Date: Friday, May 6, 2022.

Laplace Transform Table:

$f(t)$	$\mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t)f(t-c)$	$e^{-cs}\mathcal{L}\{f(t)\}$
$\delta(t-c)$	e^{-cs}
y'	$s\mathcal{L}\{y\} - y(0)$
y''	$s^2\mathcal{L}\{y\} - sy(0) - y'(0)$

1. *(10 points)* Solve the following equation and write your answer in explicit form

$$\begin{cases} y' + \cos(t)y = te^{-\sin(t)} \\ y(0) = 4 \end{cases}$$

$y =$

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2. *(10 points)* Use separation of variables to solve the following. Write your answer in explicit form.

$$\begin{cases} y' = \dfrac{y^2+1}{x^2+1} \\ y(0) = 1 \end{cases}$$

$y = \boxed{}$

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3. *(10 points)* Use undetermined coefficients to solve

$$\begin{cases} y'' + 9y = 5\cos(2t) - 10\sin(2t) \\ y(0) = 3 \\ y'(0) = 5 \end{cases}$$

$y =$

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4. *(10 points)* Use Laplace transforms to solve

$$\begin{cases} y'' - 7y' + 12y = \delta(t-3) \star \delta(t-5) \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

$y =$

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5. *(10 points)* Find a series solution of

$$\begin{cases} y'' - (2x)\, y' - 2y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Note: Write your final answer in explicit form, without series

$y = \boxed{}$

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6. *(10 points)* Solve $\mathbf{x}' = A\mathbf{x}$ with $\mathbf{x}(0) = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$ where

$$A = \begin{bmatrix} 5 & 10 \\ -2 & -3 \end{bmatrix}$$

Note: Simplify your final answer and write it as a single vector.
NO need to draw the phase portrait

$\mathbf{x}(t) =$	

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7. *(10 points)* Use variation of parameters to find a particular solution to $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ where

$$A = \begin{bmatrix} 1/t & -1 \\ 1 & 1/t \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} t \\ -t \end{bmatrix}$$

Note: Simplify your final answer and write it as a single vector. Assume the general solution of the homogeneous equation is

$$\mathbf{x_0}(t) = C_1 \begin{bmatrix} t\sin(t) \\ -t\cos(t) \end{bmatrix} + C_2 \begin{bmatrix} t\cos(t) \\ t\sin(t) \end{bmatrix}$$

$\mathbf{x_p}(t) =$

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(Scratch paper)