

MATH 308 – MOCK FINAL EXAM

Note: Please also look at the final exam review problems for a more comprehensive study experience

1. Solve and write in explicit form

$$\begin{cases} 2\frac{dy}{dx} = \frac{xy\sqrt{1+x^2}}{\ln(y)} \\ y(0) = 1 \end{cases}$$

Date: Friday, May 6, 2022.

2. Solve the following logistic differential equation

$$\begin{cases} \frac{dy}{dx} = 3y \left(1 - \frac{y}{4}\right) \\ y(0) = \frac{1}{2} \end{cases}$$

3. Use variation of parameters to find the general solution of

$$y'' + 3y' + 2y = \sin(e^t)$$

Simplify your answer

4.

$$\begin{cases} y'' + 4y = \delta(t - 3) \star (5e^t) \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

5. Solve using series and write your answer in explicit form

$$\begin{cases} y'' - xy' + 2y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

6. Use matrix exponentials to solve $\mathbf{x}' = A\mathbf{x}$ with $\mathbf{x}(0) = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$$A = \begin{bmatrix} 5 & -4 \\ 1 & 1 \end{bmatrix}$$

7. Solve $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$ using **both** undetermined coefficients and variation of parameters, where

$$A = \begin{bmatrix} -4 & 6 \\ -8 & 10 \end{bmatrix} \quad \mathbf{f} = \begin{bmatrix} 6e^{4t} \\ 8e^{4t} \end{bmatrix}$$

Assume $A = PDP^{-1}$ with $D = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$