APMA 0350 – MIDTERM

Name	
Banner ID	
Signature	

Instructions: Welcome to your midterm. You have 50 minutes to take this exam, for a total of 40 points. No books, notes, cheat sheets, calculators, or cellphones are allowed. **Please put your answers in the boxes provided.** Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on scratch paper, please check the box "Work on scratch paper"

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Brown University Academic Code.

Date: Friday, October 28, 2022.

1. *(8 points)* Find the general solution of the ODE (assume $t > 0$)

$$t^2 y' = -t(t+2)y + e^t$$

$y =$

☐ Work on Scratch Paper

2. *(8 points)* Solve the ODE and leave your answer in implicit form

$$\begin{cases} \dfrac{dy}{dx} = \dfrac{e^x \sin(y) - 2x - 4y}{4x + 2y - e^x \cos(y)} \\ y(2) = 0 \end{cases}$$

Answer:	

☐ Work on Scratch Paper

3. *(8 points)* Solve $\mathbf{x}' = A\mathbf{x}$ where

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix} \text{ where } \mathbf{x}(0) = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Do **NOT** draw a phase portrait!

Write your answer in the form $e^{at}\begin{bmatrix} c\cos(bt) + d\sin(bt) \\ f\cos(bt) + g\sin(bt) \end{bmatrix}$ with integers only.

$\mathbf{x}(t) =$	

☐ Work on Scratch Paper

4. *(8 points)* Use **matrix exponentials** to solve $\mathbf{x}' = A\mathbf{x}$ where

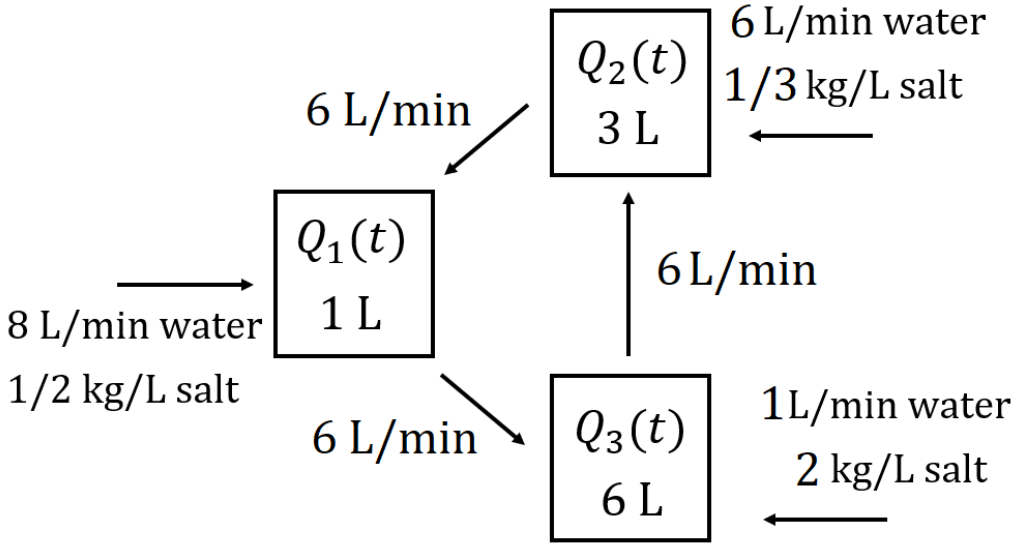
$$A = \begin{bmatrix} -1 & 4 \\ -9 & 11 \end{bmatrix} \text{ where } \mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Write your answer in the form $e^{\lambda t}\begin{bmatrix} at+b \\ ct+d \end{bmatrix}$ with integers only

$\mathbf{x}(t) =$	

☐ Work on Scratch Paper

5. *(8 points)* Consider the following configuration of chemical tanks. Assume the amount of water in each tank is constant.



Find A and $\mathbf{b}$ such that the system has the form

$$\mathbf{Q}'(t) = A\mathbf{Q}(t) + \mathbf{b} \quad \text{where } \mathbf{Q}(t) = \begin{bmatrix} Q_1(t) \\ Q_2(t) \\ Q_3(t) \end{bmatrix}$$

Here $Q_i(t)$ is the amount of salt in tank i (in kg)

No justification required

$$A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

☐ Work on Scratch Paper

(Scratch Paper)