## MATH 308 - MIDTERM 2

| Name |  |  |
| :---: | :---: | :---: |
| Student ID |  |  |
| Section | 509 | 512 |
| Signature |  |  |

Instructions: Welcome to your Midterm! You have 75 minutes to take this exam, for a total of 50 points. No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your final answer, but also on your work. Please put your answers in the boxes provided. If you need to continue your work on a scratch paper, please check the box "Work on Scratch Paper," or else your work will be discarded.

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Aggie Honor Code.

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## Laplace Transform Table:

| $f(t)$ | $\mathcal{L}\{f(t)\}$ |
| :---: | :---: |
| 1 | $\frac{1}{s}$ |
| $e^{a t}$ | $\frac{1}{s-a}$ |
| $t^{n}$ | $\frac{n!}{s^{n+1}}$ |
| $\sin (a t)$ | $\frac{a}{s^{2}+a^{2}}$ |
| $\cos (a t)$ | $\frac{s}{s^{2}+a^{2}}$ |
| $u_{c}(t)$ | $\frac{e^{-c s}}{s}$ |
| $u_{c}(t) f(t-c)$ | $e^{-c s} \mathcal{L}\{f(t)\}$ |
| $\delta(t-c)$ | $e^{-c s}$ |
| $y^{\prime}$ | $s \mathcal{L}\{y\}-y(0)$ |
| $y^{\prime \prime}$ | $s^{2} \mathcal{L}\{y\}-s y(0)-y^{\prime}(0)$ |

1. $(10=5 \times 2$ points $)$ Guess the form of the particular sol of
(a) $y^{\prime \prime}+4 y=\sin (3 t)$
(b) $y^{\prime \prime}+4 y=e^{3 t} \cos (2 t)$
(c) $y^{\prime \prime}+4 y=t \sin (2 t)$
(d) $y^{\prime \prime}+4 y=e^{2 t}$
(e) $y^{\prime \prime}+4 y=t^{3}+3$

Note: No justification required here

| $(a)$ | $y_{p}=$ |
| :--- | :--- |
| $(b)$ | $y_{p}=$ |
| $(c)$ | $y_{p}=$ |
| $(d)$ | $y_{p}=$ |
| $(e)$ | $y_{p}=$ |

2. (10 points) Use variation of parameters to find a particular solution to

$$
y^{\prime \prime}-4 y^{\prime}+4 y=\frac{e^{2 t}}{t^{2}}
$$

$y_{p}=1$
3. (10 points) Use Laplace Transforms to solve

$$
\left\{\begin{aligned}
y^{\prime \prime}-3 y^{\prime}-10 y & =14 \delta(t-4) \\
y(0) & =1 \\
y^{\prime}(0) & =-2
\end{aligned}\right.
$$

$\square$Work on Scratch Paper
4. (10 points) Find a function whose Laplace transform is

$$
\left(\frac{2 s-3}{4 s^{2}-12 s+25}\right)\left(\frac{1}{s^{2}+9}\right)
$$

Write your answer in terms of an integral

Hint: Do both terms separately
$\square$
5. (10 points) Find a recurrence relation for the coefficients $a_{n}$ in a series solution of

$$
y^{\prime \prime}+x y^{\prime}+y=0
$$

Simplify your answer

Note: You do not need to find an explicit formula for the $a_{n}$
Answer $\quad$ (
(Scratch paper)


[^0]:    Date: Monday, April 4, 2022.

