MAT 267 Spring 2021, Final Exam, Instructor: Peyam Tabrizian

Part I: Multiple Choice. Select the correct answer for each problem. Make a table of all answers at the top of the first page of your test. There are 7 problems, each worth 6 points, for a total of 42 possible points.

1. The arc length of the curve C parameterized as $\mathbf{r}(t) = \langle 2t+1, 3t-4, 5t \rangle,\ 0 \leq t \leq 3$ is

A. 3

B. $3\sqrt{17}$

C. $3\sqrt{38}$

D. 30

E. none of the above

2. If $w = x^2 + 3y^2$, $x = se^{2t}$ and $y = st^2$, which of the expressions represents $\frac{\partial w}{\partial t}$?

A. $\frac{\partial w}{\partial t} = (2x)(2e^{2t}) + (6y)(t^2)$

B. $\frac{\partial w}{\partial t} = (2x)(2se^{2t}) + (6y)(st^2)$

C. $\frac{\partial w}{\partial t} = (2x)(se^{2t}) + (6y)(2st)$

D. $\frac{\partial w}{\partial t} = (2x)(2se^{2t}) + (6y)(2st)$

E. none of the above

3. Find the directional derivative of $f(x,y,z) = x^2yz - 3xz + 4xy$ at the point $(1,0,1)$ and in the direction of $\mathbf{v} = \langle 3,4,0 \rangle$.

A. $11/\sqrt{43}$

B. $\langle -3, 5, -3 \rangle$

C. $\sqrt{43}$

D. $11/5$

E. none of the above

4. Let E be the region bounded by the two hemispheres $z = \sqrt{1-x^2-y^2}$ and $z = \sqrt{9-x^2-y^2}$ and the xy plane. The triple integral $\iiint_E \sqrt{x^2+y^2+z^2}\ dxdydz$ in spherical coordiantes is

A. $\int_0^{2\pi} \int_0^{\pi} \int_1^3 \rho^3 \sin\phi\ d\rho\ d\phi\ d\theta$

B. $\int_0^{2\pi} \int_0^{\pi/2} \int_1^3 \rho^3 \sin\phi\ d\rho\ d\phi\ d\theta$

C. $\int_0^{2\pi} \int_0^{\pi} \int_1^9 \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta$

D. $\int_0^{2\pi} \int_0^{\pi/2} \int_1^3 \rho\ d\rho\ d\phi\ d\theta$

E. none of the above

5. Find the curl of $\mathbf{F}(x,y,z) = \langle 5x-3y, xyz^2, x^2-z^2 \rangle$ at the point $(1,2,1)$.

A. $\langle -4, -2, 5 \rangle$

B. $\langle 5, 1, -2 \rangle$

C. $\langle -4, 2, -5 \rangle$

D. 4

E. none of the above

6. Find the volume of the solid region that lies under the surface $z = xy$ and over the region in the xy plane bounded by the curves $y = 2x$ and $y = x^2$.

A. $8/3$

B. $4/3$

C. $32/3$

D. 8

E. none of the above

7. Find the surface area of the parametric surface $\mathbf{r}(u,v) = \langle 3u - v, 2v, u + v \rangle,\ 0 \leq u \leq 2,\ 0 \leq v \leq 4$.

A. $8\sqrt{60}$

B. $8\sqrt{56}$

C. 8

D. 56

E. none of the above

Part II: Free Response. Solve each problem, showing all work clearly and thoroughly. Draw a box around your final answers and include units where applicable. There are 5 problems worth a total of 58 points.

1. [12 points] To the Moon!!! A toy rocket with a mass of 3 kg is launched from the ground at $t = 0$ and crashes on the ground ($z = 0$) some time later. During the flight, its position is $\mathbf{r}(t) = \langle t^4 + 3t^2, t^2 + 2t, 6t - 3t^2 \rangle$ where t is in seconds and distances are in meters.

a) [6 pts] Find the speed the rocket was traveling when it crashed. (exact answer please)

b) [6 pts] Find the force vector $\mathbf{F}$ that was acting on the rocket when it launched.

2. [12 points] An IKEA box KALLAX has dimensions x, y and z in such a way that $2x + y + z = 6$. What are the dimensions of the box that produce the largest possible volume? Don't forget to check that you actually have a local maximum.

3. [14 points] Evaluate the surface integral $\iint_S (xy + z)\ dS$ where S is the part of the plane $z = x + 3y$ that lies above the rectangle $[0, 1] \times [0, 2]$. No picture needed. Please use parametrizations, don't use the "direct formula"

4. [14 points] Find the flux of the vector field $\mathbf{F}(x, y, z) = \langle z, x, y \rangle$ through the surface S parametrized by $\mathbf{r}(u, v) = \langle 2u, 3v, u + v \rangle,\ \ 0 \le u \le 1,\ \ 0 \le v \le 2$. Assume the surface has upward orientation. No picture needed.

5. [6 points] [Juicy Peyam Special] Suppose a donut S has a parametrization in the form $r(\theta, \alpha)$ (with θ and α both between 0 and 2π) in such a way that the normal vector is

$$a(b + a\cos(\alpha))\ \langle \cos(\alpha)\cos(\theta), \cos(\alpha)\sin(\theta), \sin(\alpha) \rangle$$

where $0 < a < b$ are fixed constants. Find the surface area of S. No picture needed.