MATH 251 – FINAL EXAM

Name	
Student ID	
Section	501
Signature	

Instructions: Welcome to your Final Exam! You have 120 minutes to take this exam, for a total of 100 points. No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your final answer, but also on your work. **Please put your answers in the boxes provided.** If you need to continue your work on the back, please check the box "Work on Back," or else your work will be discarded.

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Aggie Honor Code.

Date: Monday, December 13, 2021.

1. *(10 points)* Calculate $\int_C F \cdot dr$ where $F = \frac{4}{3}\left\langle -y^3, x^3 \right\rangle$ and C is the upper half circle $x^2+y^2 \leq 9$ with $y \geq 0$, oriented **clockwise**. Simplify your answer.

Answer	

☐ Work on back

2. *(10 points)* Use parametrizations to find $\int_C F \cdot dr$, where $F = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ and C is the circle centered at $(0,0)$ and radius 2, oriented counterclockwise

Answer	

☐ Work on back

3. *(10 points)* Find $\int_C F \cdot dr$, where $F = \left\langle 2xe^{-y}, 2y - x^2e^{-y} \right\rangle$ and C is the line segment from $(1, 0)$ to $(3, 5)$

Answer	

☐ Work on back

4. *(15 points)* Find $\iint_S 3\sqrt{x+2y+2z}\,dS$, where S is the surface parametrized by:

$$\begin{cases} r(u,v) = \left\langle u^2, uv, \frac{1}{2}v^2 \right\rangle \\ \quad 0 \leq u \leq 1 \\ \quad 0 \leq v \leq 2 \end{cases}$$

Answer	

☐ Work on back

5. *(15 = 10 + 5 points)*

(a) Find a such that $F = \operatorname{curl}(G)$, where $F = \langle 0, 0, -3x \rangle$ and $G = \left\langle xy, ax^2, z \right\rangle$

(b) Use (a) to calculate $\iint_S F \cdot d\mathbf{S}$ where $F = \langle 0, 0, -3x \rangle$ and S is the paraboloid $z = 25 - x^2 - y^2$ above the plane $z = 9$, oriented upwards

Answer for (a)	
Answer for (b)	

☐ Work on back

6. *(15 points)* Find $\iint_S F \cdot d\mathbf{S}$, where $F = \left\langle 3x, -y + e^{x(z^2)}, \sin(xy) \right\rangle$ and S is the boundary of the region E in the first octant bounded by $y = 1 - x^2$ and $y + z = 2$, oriented outwards.

Answer	

☐ Work on back

7. *(15 = 5 + 10 points)* Let $F = \langle x, \sin(\sin(y)), \cos(\cos(z)) \rangle$

(a) Show F is conservative

(b) Calculate $\int_C F \cdot dr$, where C is the curve parametrized by $r(t) = \langle t, \sin(t), \sin(t) \rangle$, $0 \leq t \leq \pi$.

Hint: Since $\int_C F \cdot dr$ is independent of path, can you think of a simpler path connecting the start and endpoints?

Answer for (a)	(show work)
Answer for (b)	

☐ Work on back

8. *(10 = 5 + 5 points)* The two parts of this problem are independent of each other

   (a) Calculate $\iint_S \operatorname{curl}(F) \cdot d\mathbf{S}$ where $F = \left\langle x^3, y^4, z^5 \right\rangle$ and S is the sphere centered at $(0, 0, 0)$ and radius 3

   (b) Is there a vector field G such that $F = \operatorname{curl} G$ where $F = \langle z, y, x \rangle$? Why or why not?

Answer for (a)	
Answer for (b)	

☐ Work on back