MATH 251 – FINAL EXAM

Name	
Student ID	
Section	512
Signature	

Instructions: Welcome to your Final Exam! You have 120 minutes to take this exam, for a total of 100 points. No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your final answer, but also on your work. **Please put your answers in the boxes provided.** If you need to continue your work on a back, please check the box "Work on back," or else your work will be discarded.

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Aggie Honor Code.

Date: Tuesday, December 14, 2021.

1. *(10 points)* Find the area of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Answer	

☐ Work on back

2. *(10 points)* Calculate $\int_C F \cdot dr$, where $F = \langle z^2, x^2, y^2 \rangle$ and C is the line segment from $(1, 0, 0)$ to $(4, 1, 2)$

Answer	

☐ Work on back

3. *(10 points)* Find $\int_C F \cdot dr$, where $F = \langle y, x + z\cos(y), \sin(y)\rangle$ and C is the curve parametrized by $r(t) = \langle t, \cos(t), \sin(t)\rangle$, $0 \leq t \leq \pi$

Answer	

☐ Work on back

4. *(10 points)* Find $\iint_S F \cdot d\mathbf{S}$, where $F = \left\langle -x, y, z^3 \right\rangle$ and S is the surface (oriented upwards), parametrized by:

$$\begin{cases} r(u,v) = \langle u\cos(v), u\sin(v), u\rangle \\ \qquad 1 \leq u \leq 2 \\ \qquad 0 \leq v \leq 2\pi \end{cases}$$

Answer	

☐ Work on back

5. *(15 points)* Calculate $\int_C F \cdot dr$, where

$$F = \left\langle x + y^2, y + z^2, z + x^2 \right\rangle$$

And C is the triangle with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, oriented counterclockwise from above.

Hint: C lies on a plane

Answer	

☐ Work on back

6. *(15 points)* Calculate $\iint_S \mathrm{curl}(F) \cdot d\mathbf{S}$, where

$$F = \left\langle z^3 + y^3, x^3 + z^3, y^3 + x^3 \right\rangle$$

And S is the part of the sphere $x^2 + y^2 + z^2 = 25$ strictly above the plane $z = 4$, oriented upwards

Answer	

☐ Work on back

7. *(20 points)* Find $\iint_S F \cdot d\mathbf{S}$ where

$$F = \left\langle x + z, x + 2y, x^2 \right\rangle$$

And S is part of the surface $z = 25 - x^2 - y^2$ strictly above the plane $z = 9$, oriented upwards

Warning: S is not closed

Answer	

☐ Work on back

8. *(10 points)* Find the surface area of the (delicious) surface S that has a parametrization of the form $r(u,v)$ with $0 \leq u \leq 2\pi$ and $0 \leq v \leq 2\pi$ whose normal vector is

$$\hat{n} = a(b + a\cos(v))\,\langle \cos(v)\cos(u), \cos(v)\sin(u), \sin(v)\rangle$$

Where $0 < a < b$ are fixed constants.

You don't need to find $r(u,v)$ to solve this problem.

Answer	

☐ Work on back