MATH 2E – FINAL EXAM

Name: ______________________

Student ID: ______________________

Instructions: This is it, your final hurdle to freedom! You have 120 minutes to take this exam, for a total of 100 points. No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. May your luck be orientable, and happy holidays! :)

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to section 102.1 of the UCI Student Code of Conduct.

Signature: ______________________________

1		10
2		10
3		10
4		10
5		10
6		15
7		15
8		5
9		15
Total		100

Date: Friday, December 14, 2018.

Useful formulas

Spherical coordinates:

$$
\begin{aligned}
x =& \rho \sin(\phi) \cos(\theta) \\
y =& \rho \sin(\phi) \sin(\theta) \\
z =& \rho \cos(\phi) \\
\text{Jac} =& \rho^2 \sin(\phi)
\end{aligned}
$$

1. *(10 points)* Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the sphere of radius 2 centered at $(0, 0, 0)$, oriented outwards, and $\mathbf{F} = \langle y^2 x, z^2 y, x^2 z \rangle$. Include a picture of S and its orientation.

2. *(10 points)* Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle x - y^3, 2x + x^3 \rangle$, where C is the circle centered at $(0,0)$ and radius 2, oriented counterclockwise.

3. *(10 points)* Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ with $1 \leq t \leq 2$, and $\mathbf{F} = \langle yze^{xyz}, xze^{xyz}, xye^{xyz} \rangle$.

4. *(10 points)* Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle xy, yz, xz \rangle$, and C (oriented counterclockwise) is the curve of intersection of the surfaces $x^2 + y^2 - z^2 = 9$ and $z = 4$. Include a picture of C and the surfaces.

5. *(10 points)* Let D be the parallelogram with vertices $(-2,-3),(0,-1),(1,2),(-1,0)$. Draw a picture of D and calculate

$$\iint_D (y-3x)^2(y-x)^4 dxdy$$

6. *(15 points)* Let S be the part of the cone with parametric equations $\mathbf{r}(u, v) = \langle u\cos(v), u\sin(v), u\rangle$, $0 \leq u \leq 2$, $0 \leq v \leq 2\pi$, oriented upwards (no need to draw a picture).

   (a) *(5 points)* Find the equation of the tangent plane to S at the point $(\sqrt{3}, 1, 2)$

(b) *(5 points)* Calculate $\iint_S \left(x^2 + y^2\right)^2 dS$, where S is the surface in (a).

(c) *(5 points)* Calculate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = \langle z, y, x \rangle$ and S is the surface in (a).

7. *(15 points)*

(a) *(5 points)* Find constants a and b such that $\mathbf{G} = \text{curl}(\mathbf{F})$, where $\mathbf{G} = \langle x^2e^z, 0, -2xe^z \rangle$ and $\mathbf{F} = \langle axye^z, 0, bx^2ye^z \rangle$

(b) *(10 points)* Use your answer in (a) to calculate $\iint_S \mathbf{G} \cdot d\mathbf{S}$, where S is the part of the surface $-x^2 - y^2 + z^2 = 1$ with $1 \leq z < 2$ (without the top) and $\mathbf{G} = \langle x^2 e^z, 0, -2xe^z \rangle$. Assume S is oriented in such a way that the boundary curve C is counterclockwise. Include a picture of S and C (but no need to label the orientation).

8. *(5 points)* The Pre-Finale...

Let $\mathbf{F}$ be a vector field, and let S be the sphere centered at $(0, 0, 0)$ and radius 2, oriented outwards. Calculate $\iint_S \mathrm{curl}(\mathbf{F}) \cdot d\mathbf{S}$

9. *(15 points)* ...and the Grand Finale!

(a) *(5 points)*

Definition: If $\mathbf{F} = \langle P, Q, R \rangle$ is a vector field and $f = f(x, y, z)$ is a function, then $f\ \mathbf{F}$ is the vector field $\langle fP, fQ, fR \rangle$.

Show that for any vector field $\mathbf{F}$ and any function f,

$$\operatorname{div}(f\ \mathbf{F}) = f\ \ (\operatorname{div}(\mathbf{F})) + (\nabla f) \cdot \mathbf{F}$$

(b) *(10 points)* Suppose that $f = f(x, y, z)$ solves Laplace's equation $\Delta f = 0$ in B (the book writes this as $\nabla^2 f = 0$), where B is the ball of radius R centered at $(0, 0, 0)$ (R is a constant). Moreover, suppose that $f = 0$ on the sphere of radius R centered at $(0, 0, 0)$. Use (a) with a special choice of $\mathbf{F}$ to show that $f = 0$ in B.