

## MATH 2E – FINAL EXAM

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

**Instructions:** This is it, your final hurdle to freedom! You have 120 minutes to take this exam **plus** an extra 20 minutes to upload your answers on Canvas. Please upload your answers as a **single** pdf file, and start each problem on a new page. No books, notes, calculators, cellphones, or collaborations are allowed. Remember that you are not only graded on your final answer, but also on your work. May your luck be conservative, and stay safe!

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

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*Date:* Friday, March 20, 2020.

**Useful formulas**

Spherical coordinates:

$$x = \rho \sin(\phi) \cos(\theta)$$

$$y = \rho \sin(\phi) \sin(\theta)$$

$$z = \rho \cos(\phi)$$

$$\text{Jac} = \rho^2 \sin(\phi)$$

1. (*10 points*) Failure to do this problem results in an automatic F in the course.

Please **copy** the following statement **by hand** on your sheet of paper, and name, sign, and date it:

I certify that all the work is my own and that I did not use any outside resources to complete this exam. I understand that any form of cheating results in an automatic F in the course and will be subject to further disciplinary consequences, such as academic probation or suspension from the university.

**Full Name:**

**Date:**

**Signature:**

2. (10 points) Calculate

$$\int \int \int_E (x^2 + y^2 + z^2)^2 dx dy dz$$

Where  $E$  is the region inside the surface  $x^2 + y^2 + z^2 \leq 4$  and above the surface  $z = \sqrt{x^2 + y^2}$

Include a picture of  $E$ .

**Note:** You do not need to simplify your final answer, as long as there's no more cos or sin.

3. (10 points) Calculate

$$\int_C F \cdot dr$$

Where  $F = \langle \sin(x) + y^2x, x^2y + 2x \rangle$

And  $C$  is any circle of radius 3, in the counterclockwise direction

4. (10 points) Calculate

$$\int_C F \cdot dr$$

Where  $F = \langle y, z \cos(y) + x, \sin(y) \rangle$

And  $C$  is any curve from  $(1, 1, 0)$  to  $(0, 2, 1)$ .

5. (10 points) Find

$$\iint_S F \cdot d\mathbf{S}$$

Where  $F = \langle 1, z, x^2 \rangle$

And  $S$  is the surface parametrized by

$$\begin{aligned} r(u, v) &= \langle v, uv, u + v \rangle \\ 0 &\leq u \leq 2 \\ 0 &\leq v \leq 4 \end{aligned}$$

With upward orientation (no need to draw  $S$ )

6. (10 points) Use the following change of variables to calculate

$$\int \int_D (9x^2 + 4y^2)^{\frac{5}{2}} dx dy$$

Where  $D$  is the ellipsoid  $9x^2 + 4y^2 = 1$  in the first quadrant.

$$\begin{cases} u = 3x \\ v = 2y \end{cases}$$

Include a picture of  $D$  and the transformed region  $D'$ .

7. (10 points) Calculate

$$\iint_S \operatorname{curl}(F) \cdot d\mathbf{S}$$

Where  $F = \langle x + y, y + e^z, y^5 \rangle$

And  $S$  is the portion of the sphere  $x^2 + y^2 + z^2 = 25$  *strictly* between the planes  $z = 3$  and  $z = -4$  (without the top and without the bottom), oriented outwards

Include a picture of  $S$  and its orientation.

8. (10 points) Calculate

$$\iint_S F \cdot d\mathbf{S}$$

Where  $F = \langle x + \sin(y), y + \sin(z), z + \sin(x) \rangle$

And  $S$  is the boundary of the region enclosed by  $x^2 + z^2 = 4$ ,  $y = -1$ , and  $y + z = 3$ , oriented outwards (including the top/bottom/sides)

Include a picture of  $S$  and its orientation.

8. (10 points) Calculate

$$\int_C F \cdot dr$$

Where  $F = \langle 1, x + yz, xy - \sqrt{z} \rangle$

And  $C$  is the curve parametrized by

$$r(t) = \langle 2 \cos(t), 2 \sin(t), 8 \cos(t) \sin(t) \rangle \quad 0 \leq t \leq 2\pi$$

**Hint:**  $C$  lies on the surface  $z = 2xy$

No need to draw a picture.

10. (10 points, 5 points each) The two parts are independent of each other

(a) Suppose  $f(x, y)$  satisfies  $f_{xx} + f_{yy} = 0$

Let  $C$  be any circle of radius 2, oriented counterclockwise.

Calculate:

$$\int_C (f_y) dx - (f_x) dy$$

- (b) Let  $S$  be any closed surface, oriented outwards, and let  $E$  be the inside of  $S$ .

Suppose  $f(x, y, z, t)$  satisfies  $f_{tt} = \Delta f$  in  $E$  and  $\nabla f = 0$  on  $S$ .

$$\text{Let } M(t) = \int \int \int_E f(x, y, z, t) dx dy dz$$

Show  $M_{tt}(t) = 0$

**Note:** Here  $\Delta f = f_{xx} + f_{yy} + f_{zz}$  and  $\nabla f = \langle f_x, f_y, f_z \rangle$ , and  $M_{tt}$  is the second derivative of  $M(t)$  with respect to  $t$ .