MATH 2E – FINAL EXAM

Name: ________________________

Student ID: ____________________

Instructions: This is it, your final hurdle to freedom! You have 120 minutes to take this exam **plus** an extra 20 minutes to upload your answers on Canvas. Please upload your answers as a **single** pdf file, and start each problem on a new page. No books, notes, calculators, cellphones, or collaborations are allowed. Remember that you are not only graded on your final answer, but also on your work. May your luck be conservative, and stay safe!

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

Date: Monday, March 16, 2020.

Useful formulas

Spherical coordinates:

$$\begin{aligned}
x =& \rho \sin(\phi) \cos(\theta) \\
y =& \rho \sin(\phi) \sin(\theta) \\
z =& \rho \cos(\phi) \\
\text{Jac} =& \rho^2 \sin(\phi)
\end{aligned}$$

1. *(10 points)* Failure to do this problem results in an automatic F in the course.

   Please **copy** the following statement **by hand** on your sheet of paper, and name, sign, and date it:

   I certify that all the work is my own and that I did not use any outside resources to complete this exam. I understand that any form of cheating results in an automatic F in the course and will be subject to further disciplinary consequences, such as academic probation or suspension from the university.

   Full Name:

   Date:

   Signature:

2. *(10 points)* Calculate

$$\int_C F \cdot dr$$

Where $F = \left\langle xy, y^3 \right\rangle$

C is the polygon connecting the points $(0,0), (3,0), (3,3), (0,6)$, oriented counterclockwise.

Include a picture of C and its orientation.

3. *(10 points)* Calculate

$$\int\int_S \text{curl}(F) \cdot d\mathbf{S}$$

Where $F = \left\langle x + z, x + y, x^3 \right\rangle$

And S is the portion of the surface $z = 25 - x^2 - y^2$ strictly above the plane $z = 9$ (without the bottom), oriented upwards.

Include a picture of S.

4. *(10 points)* Calculate

$$\int\int_S F \cdot d\mathbf{S}$$

Where $F = \left\langle x, y + z^3, \cos(z) \right\rangle$

S is the portion of the surface $x^2+y^2 = 4$ in the region $0 < z \leq \frac{\pi}{2}$ oriented outwards (with the top but without the bottom).

Include a picture of S and its orientation.

5. *(10 points)* Calculate

$$\int\int\int_E e^{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}} dxdydz$$

Where E is the region inside the surface $x^2 + y^2 + z^2 \leq 4$ in the first octant.

Include a picture of E.

6. *(10 points)* Use **parametrizations** to find

$$\int\int_S z^2 \ dS$$

Where S is the portion of the surface $z^2 = x^2 + y^2$ strictly between $z = 1$ and $z = 2$ (without the top or the bottom)

Include a picture of S

7. *(10 points)* Calculate

$$\int_C F \cdot dr$$

Where $F = \left\langle 2xz + y^2, 2xy, x^2 + 3z^2 \right\rangle$

And C is any curve going from $(1, 1, 0)$ to $(1, 0, 1)$.

8. *(10 points)* Calculate

$$\int\int_D (y+x)e^{(y-x)(y+x)}dxdy$$

Where D is the region between the lines $y = x$, $y = x + 2$, $y = -x$ and $y = -x + 3$.

Include a picture of D.

9. *(10 points)* Calculate

$$\int_C F \cdot dr$$

Where $F = \left\langle e^{x^2}, x + y^2 + z, y + \sin(z) \right\rangle$

And C is the triangle connecting the points $(2, 0, 0), (0, 1, 0), (0, 0, 1)$, in the counterclockwise direction.

Include a picture of C

Hint: C lies on the plane with equation $x + 2y + 2z = 2$.

10. *(10 points)* Use the adult version of the surface integral (the one that involves the unit normal vector **n**) to calculate

$$\int\int_S F \cdot d\mathbf{S}$$

$$F = \left\langle \frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}}, \frac{y}{(x^2+y^2+z^2)^{\frac{3}{2}}}, \frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}} \right\rangle$$

And S is the sphere centered at $(0,0,0)$ and radius r (where r is a positive constant), oriented outwards.

Do **NOT** use parametrizations or spherical coordinates