FIVE FTC

1. Fundamental Theorem of Calculus

FTC:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

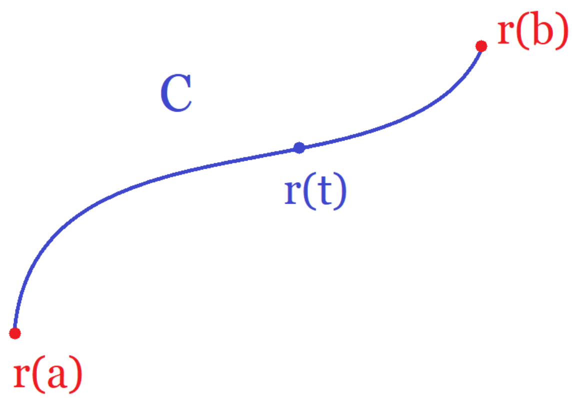
This says: $\int$ Derivative = Function, where a and b are the endpoints (boundary) of $[a, b]$

2. FTC for Line Integrals

FTC for Line Integrals:

$$\int_C \nabla f \cdot dr = f \text{ (end) } - f \text{ (start)}$$

Date: Thursday, December 9, 2021.



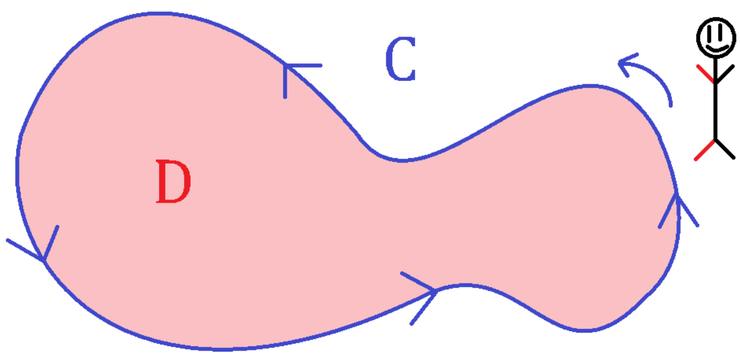
Direct analog of FTC, but for line integrals

Explains why conservative vector fields ($F = \nabla f$) are nice

3. Green's Theorem

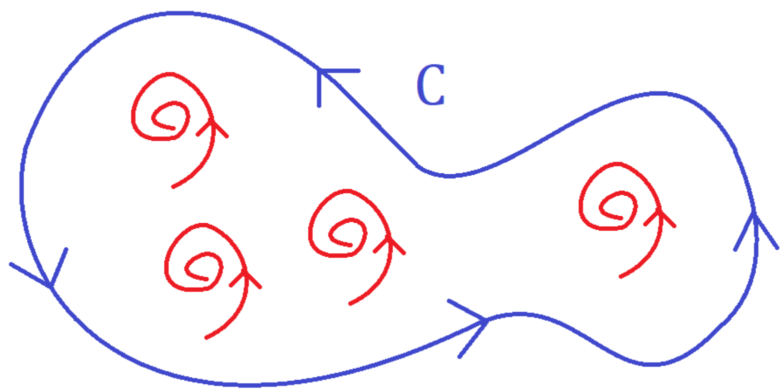
Green's Theorem:

$$\int_C F \cdot dr = \int\int_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dxdy \qquad F = \langle P, Q \rangle$$



Interpretation:

$$\underbrace{\int_C F \cdot dr}_{\text{Macro Rotation}} = \underbrace{\int\int_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dxdy}_{\text{Sum of Micro Rotations}}$$



Only works in 2 dimensions

Only works if C is closed

Basically the only tool for non-conservative F in 2 dimensions

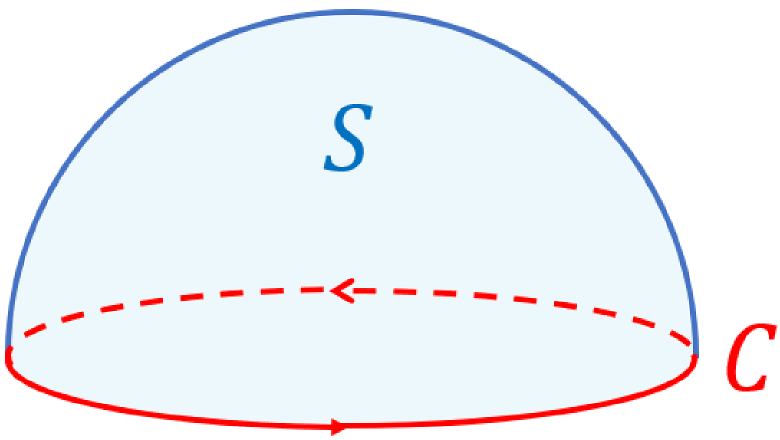
Make sure D is to your Left (Wa**L**k **L**eft)

Again, notice this says $\int\int_D$ Derivatives $= \int_C$ Function , where $C =$ **Boundary** of D

4. Stokes' Theorem

Stokes' Theorem:

$$\int_C F \cdot dr = \int\int_S \text{curl}(F) \cdot d\mathbf{S}$$



Interpretation:

$$\underbrace{\int_C F \cdot dr}_{\text{Macro Rotation}} = \underbrace{\int\int_S \text{curl}(F) \cdot d\mathbf{S}}_{\text{Sum of Micro Rotations}}$$

$3D$ analog of Green

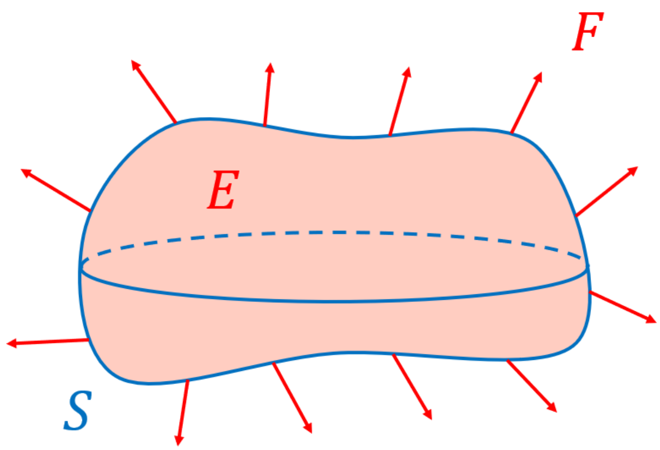
Make sure S is to your **L**eft (Wa**L**k **L**eft)

This says $\int\int_S$ Derivatives $= \int_C$ Function , $C =$ **Boundary** of S

5. Divergence Theorem

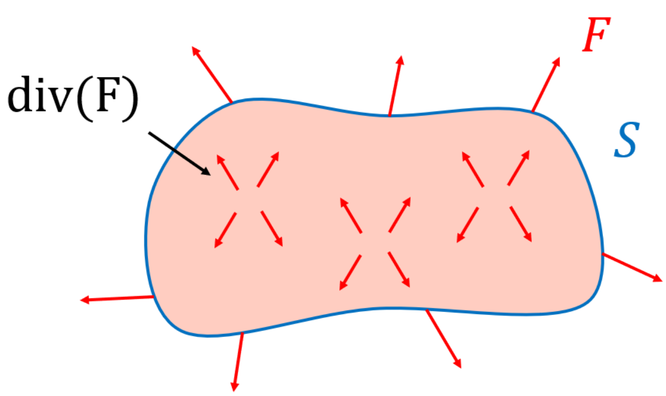
Divergence Theorem:

$$\int\int_S F \cdot d\mathbf{S} = \int\int\int_E \operatorname{div}(F)\, dxdydz$$



Interpretation:

$$\underbrace{\int\int_S F \cdot d\mathbf{S}}_{\text{Net Flux}} = \underbrace{\int\int\int_E \operatorname{div}(F)\, dxdydz}_{\text{Sum of Expansions of } F}$$



Only works for **closed** S (otherwise need to close it)

Very convenient, essentially the only tool for non-conservative F in $3D$

Make sure S is oriented outwards

This says $\int\int\int_E \text{ Derivatives } = \int\int_S \text{ Function }$, $S =$ **Boundary** of E