MATH 409 – HOMEWORK 1

General Rules:

- Homework is generally due on Fridays at 11:59 pm. You submit it by scanning your answers using a scanning app (like CamScanner or Adobe Scan) and uploading them to Canvas.
- Hints and solutions to the book problems can be found in the back of the book
- AP refers to the Additional Problems below. Please do them, as they are fair game to be graded
- This assignment is shorter than usual because you have less time to do it. Expect to roughly have 12 problems per HW set
- It's *completely* normal if you're struggling with the homework and spending lots of time on it. This is analysis, a class that is by its nature very hard. Don't lose hope, remember what doesn't kill you makes you stronger! ☺ And of course, please feel free to ask for help whenever you need.

Reading: Section 1. You don't need to memorize the axioms of $\mathbb{N}$, but definitely understand them. Make sure you understand the paragraph on page 2 that starts with "Here is another way to view axiom N5."

Date: Due: Friday, September 3, 2021.

- **Section 1:** 9, 11, AP1, AP2, AP3 (Optional: AP4)

Additional Problem 1: Prove by induction on $n \geq 0$ that

$$\int_0^\infty x^n e^{-x} dx = n!$$

You may assume without proof that $\lim_{x\to\infty} x^k e^{-x} = 0$ for any $k > 0$

Additional Problem 2: Prove by induction on $n \geq 1$ that

$$\sin(x) + \sin(3x) + \cdots + \sin((2n-1)x) = \frac{1 - \cos(2nx)}{2\sin(x)}$$

Additional Problem 3: Prove or disprove[1] that $|\cos(nx)| \leq n\,|\cos(x)|$ for all natural numbers n and all real numbers x

OPTIONAL Additional Problem 4: Here is how to actually construct $\mathbb{N}$. Define $0 = \emptyset$ (the empty set) and for every n, let $n + 1 = n \cup \{n\}$. Use this definition to calculate $1, 2, 3$ (Note we can then define $a < b$ simply as $a \in b$)[2]

[1] As a reminder, *Proving* means showing it's true for all n and x, and *Disproving* means finding a *specific* n and a *specific* x that makes the statement false (or doing a proof by contradiction)

[2] If you're interested in this, I'd highly recommend you to take a course in set theory and check out this book: Elements of Set Theory