HOMEWORK 1 – SELECTED BOOK SOLUTIONS

1.9(b): Let P_n be the proposition $2^n > n^2$

Base Case: $n = 5$

$$2^5 = 32 > 25 = 5^2 \checkmark$$

Inductive Step: Suppose P_n is true, that is $2^n > n^2$

Show P_{n+1} is true, that is $2^{n+1} > (n+1)^2$, but:

$$2^{n+1} = 2\left(2^n\right) > 2n^2$$

(where in the last step we used the inductive hypothesis)

Now $2n^2 > (n+1)^2$ is equivalent to $2n^2 > n^2 + 2n + 1$, which is equivalent to $n^2 - 2n - 1 > 0$

However:

$$n^2 - 2n - 1 = n^2 - 2n + 1 - 2 = (n+1)^2 - 2 \geq (5+1)^2 - 2 = 34 > 0$$

where we used the fact that $n \geq 5$

Therefore $2^{n+1} > (n+1)^2$ so P_{n+1} is true, and therefore P_n is true for all $n \geq 5$, that is

$$2^n > n^2 \quad \square$$

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