MATH S4062 – HOMEWORK 1

Note: The problems refer to the Rudin textbook. See below for hints

- **Chapter 7:** 2, 7

Note: For 2 you're allowed to use problem 1 (without proof)

Please **also** do the additional problems below.

Additional Problem 1: Consider the following sequence of functions f_n on $[0, 1]$, sometimes called the **growing steeple**

$$f_n(x) = \begin{cases} nx & \text{if } 0 \leq x \leq \frac{1}{n} \\ 2 - nx & \text{if } \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0 & \text{if } \frac{2}{n} \leq x \leq 1 \end{cases}$$

Show that f_n converges pointwise to 0, but not uniformly to 0.

Additional Problem 2: Suppose $f_n : \mathbb{R} \to \mathbb{R}$ is a sequence of uniformly continuous functions and $f_n \to f$ uniformly as $n \to \infty$. Show that f is uniformly continuous.

Date: Due: Friday, July 8, 2022.

Hints:

Problem 2: this is similar to the proof that $f + g$ and fg are continuous.

Problem 7: First show that $|f_n(x)| \leq \frac{1}{2\sqrt{n}}$. In order to do this, use that $a^2 + b^2 \geq 2ab$ (which follows because $(a - b)^2 \geq 0$)

Additional Problem 1: Write down the negation of the definition of uniform convergence. I think $\epsilon = \frac{1}{2}$ should work

Additional Problem 2: This is similar to the proof with continuity.