MATH S4062 – HOMEWORK 2

- **Chapter 7:** 6, 15, 18

Please **also** do the additional problems below.

Additional Problem 1: Find an example of a function $f : \mathbb{R} \to \mathbb{R}$ such that $|f'(x)| < 1$ for all x, but f has no fixed point. (I recommend playing around with exponential functions before looking at the hint)

Additional Problem 2: Show that the sequence f_n defined by

$$f_n(x) = \cos(x + n) + \ln\left(1 + \frac{\sin(nx)}{\sqrt{n + 2}}\right)$$

Is equicontinuous on $[0, 2\pi]$

Hints:

Problem 15: f must be constant on $[0, \infty)$. Suppose there are $x \neq y$ non-negative with $f(x) \neq f(y)$, let $\epsilon = |f(x) - f(y)|$, and go from there.

Additional Problem 1: Try out $f(x) = \ln(e^x + 1)$ (thanks Peiyan for the suggestion!) or $f(x) = x + \frac{1}{e^x + 1}$

Additional Problem 2: For the second function, use a theorem in lecture that gives a sufficient condition for equicontinuity. Also the inequality $\ln(1 + x) \leq x$ might be useful.