MATH S4062 – HOMEWORK 7

- **Chapter 9:** 16, 17(b)(c), 19, 23

Please also do the Additional Problem below:

Additional Problem 1: Show that the Implicit Function Theorem implies (the following version of) the Inverse Function Theorem:

Inverse Function Theorem: Suppose $f : \mathbb{R}^n \to \mathbb{R}^n$ is C^1. If $\det(f'(a)) \neq 0$ for some a, there is an open neighborhood U of a and an open neighborhood V of $f(a)$ and $g : V \to U$ such that $f(g(y)) = y$ for all y. Moreover

$$g'(f(a)) = (f'(a))^{-1}$$

Note: Hence, combined with the proof given in lecture, the Inverse and Implicit function theorems are equivalent.

Date: Due: Tuesday, August 2, 2022.

Hints:

Problem 16: For the last part, it's useful to notice that $f'\left(\frac{1}{\pi m}\right) = 1 - 2(-1)^m$ so the minimum value of f inside $\left[\frac{1}{(2m+1)\pi}, \frac{1}{(2m)\pi}\right]$ is assumed at an interior point.

Problem 17: The Jacobian is just the determinant of the derivative matrix and f is not one-to-one by periodicity in y. For (c), it's useful to let $u = e^x \cos(y)$ and $v = e^x \sin(y)$ and solve for x and y in terms of u and v

Problem 19: **Please use the Implicit Function Theorem for the first three parts.** For the last part, if you add the last two equations and subtract it from the first, you should get $u = 0$ or $u = 3$, so generally the system cannot be solved for x, y, z in terms of u.

Problem 23: **Please do this using the Implicit Function Theorem**

Additional Problem 1: Apply the Implicit Function theorem to $F : \mathbb{R}^{n+n} \to \mathbb{R}^n$ given by $F(x, y) = y - f(x)$ and $(x_0, y_0) = (a, f(a))$ and let $g(y) = G(y)$, where G is from the Implicit Function Theorem. Careful that here the roles of x and y are switched