LECTURE 1: 3D COORDINATE SYSTEMS

1. Introduction

Hello everyone and welcome to Math 251, fun in several variables! My name is Peyam and I'll be your instructor this semester

- **Logistics:** All the info is on the syllabus, which can be found on Howdy
- **Office Hours:** W 3−4:30 pm and Th 2:15−3:45 pm via Zoom. Please come, I'd be happy to help! You are allowed to request in person office hours.
- **Textbook:** *Calculus: Early Transcendentals* (8nd edition) by Stewart.
- **Resources:** Here are some resources you can use:
  - Course Website: For the lecture notes, YouTube videos, and practice exams. NOT set up yet, so in the meantime everything is posted on Canvas
  - Canvas Check your grades and take your quizzes
  - WebAssign To do your homework
  - Campuswire A really cool forum; use it to ask questions
  - YouTube Channel My YouTube channel, for videos related to this course

Date: Monday, August 30, 2021.

- TikTok Channel For other fun videos

- **Grading:**

  - **HW 10 %**, due every Friday by 11:55 pm on WebAssign, with the exception of Exam Weeks and Thanksgiving. The first HW is due this Friday

  - **Quizzes 10 %**, due every Friday by 11:55 pm on Canvas, except for Exam Weeks and Thanksgiving. They will usually be posted on Friday morning and you'll upload your answers on Canvas. The only exception is the first quiz, which will be in-class this Friday. The lowest 2 quizzes are dropped

  - **Midterms 20 % each**. 3 Midterms, during class, on the following days:

    Midterm 1: Friday, Sep 24
    Midterm 2: Friday, Oct 15
    Midterm 3: Friday, Nov 12

  - **Final 20 %**, not cumulative, only covers Chapter 16

    501 Final Exam: Monday, Dec 13, 10:30 am to 12:30 pm
    512 Final Exam: Tuesday, Dec 14, 3:30 pm to 5:30 pm

  - **Extra Credit 1 %**: Given to the top posters on Campuswire

- **Grades:** You will be graded according to the scale in the syllabus, so everyone can get an A if they work hard. I will try my best to be as generous as I can

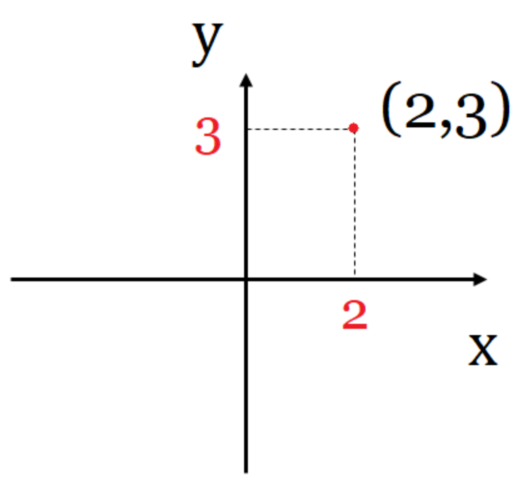
- **Finally:** Sit back, relax, and enjoy the course. If you don't like multivariable calculus, I completely understand. When I took this class in freshman year, I was constantly confused, and didn't even ace the class! But when I first taught this course in Fall 2018, I completely fell in love with it, and my goal this semester is to share this love with you ☺

2. 3D Coordinate Systems

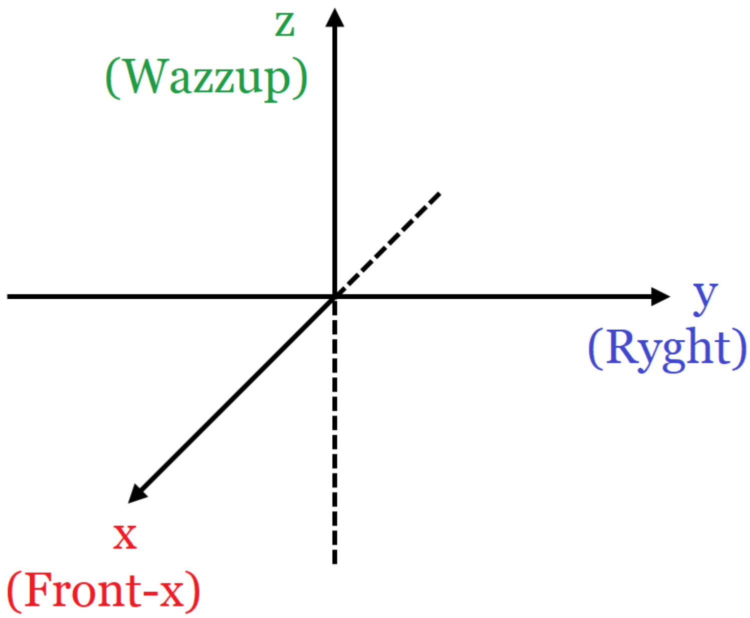
So far in your calculus adventure, you have dealt with 2 dimensions, where points with have two components, x and y

Example 1:

Plot the point $(2, 3)$



The only problem is that we don't live in 2 dimensions, but in 3 dimensions. In this course, we will crush the two-dimensional paradigm and enter the world of multi-variable calculus, also known as calculus in 3 dimensions! We will use the following convention



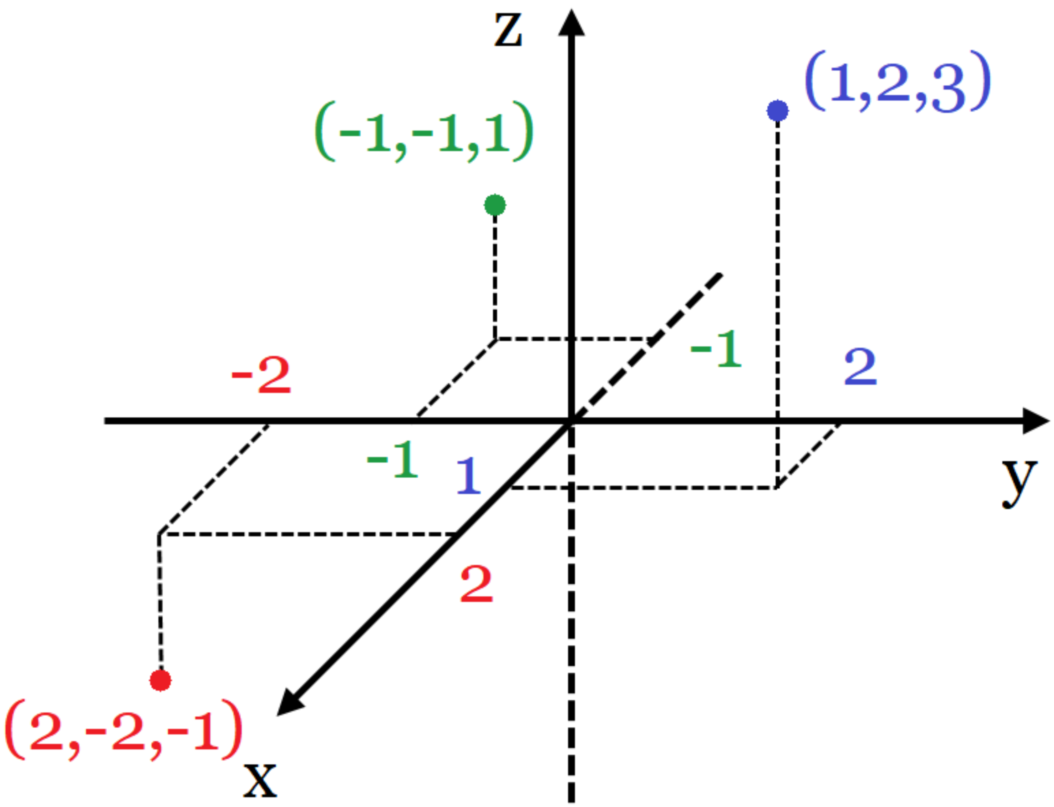
Example 2:

Plot the following points:

(a) $(1, 2, 3)$

(b) $(2, -2, -1)$

(c) (extra practice) $(-1, -1, 1)$



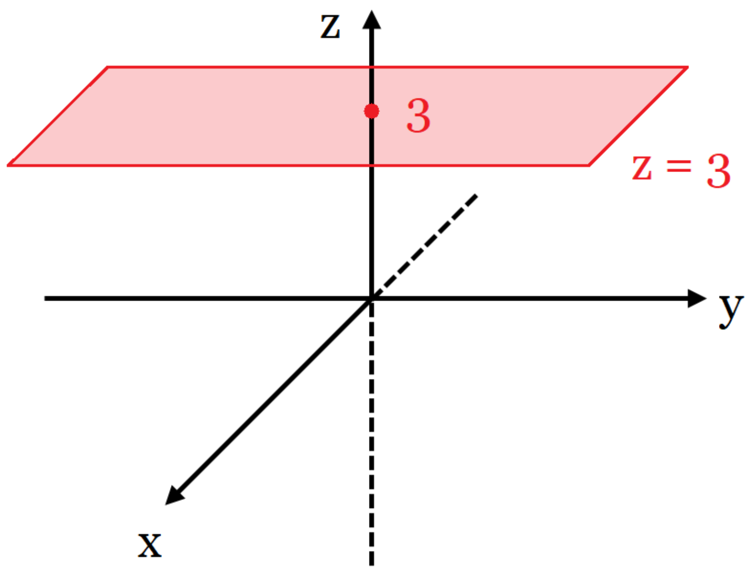
3. Basic Surfaces

Just like in 2 dimensions, where we have lines like $y = 2$, in 3 dimensions, we also have some basic surfaces such as $z = 3$:

Example 3:

Sketch the surface $z = 3$

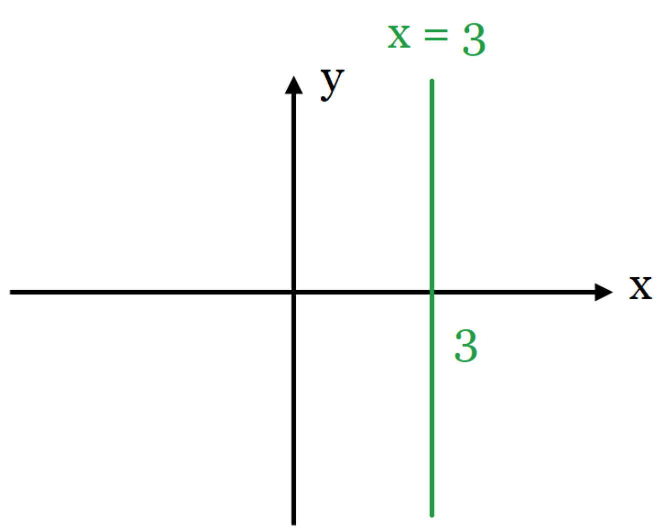
It's all the points whose height is 3, and so a (flat) plane with height 3



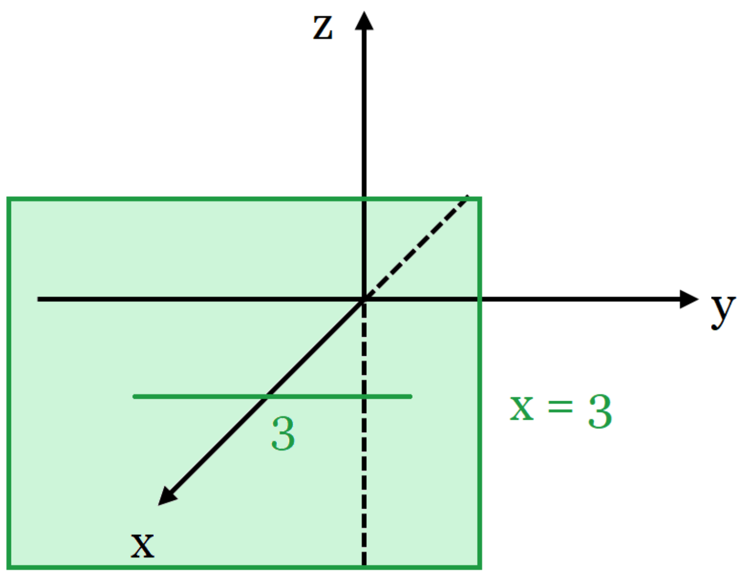
Example 4:

Sketch the surface $x = 3$

In 2 dimensions, $x = 3$ is a line which crosses the $x-$axis at 3.



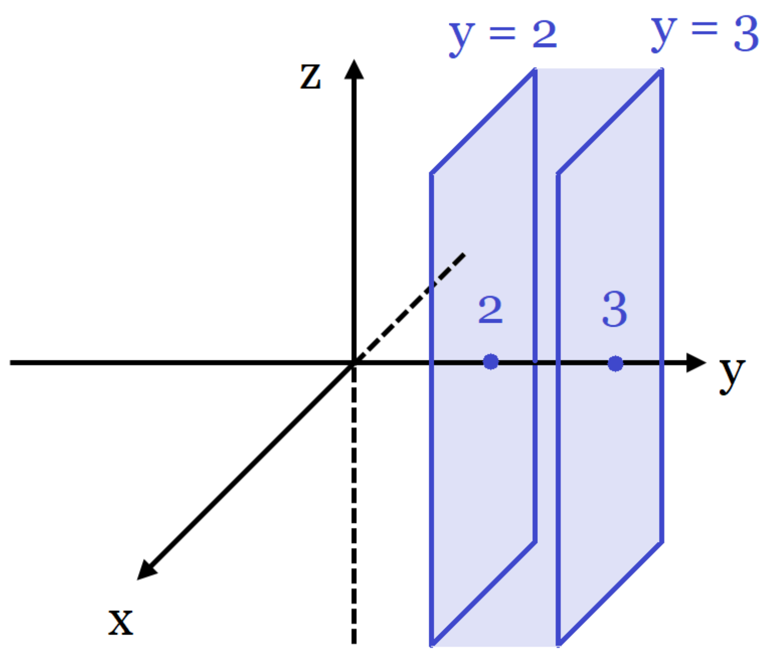
Same thing here, except it's a *plane* that crosses the $x-$axis at 3 (it looks like a sheet that is parallel to what's called the yz-plane)



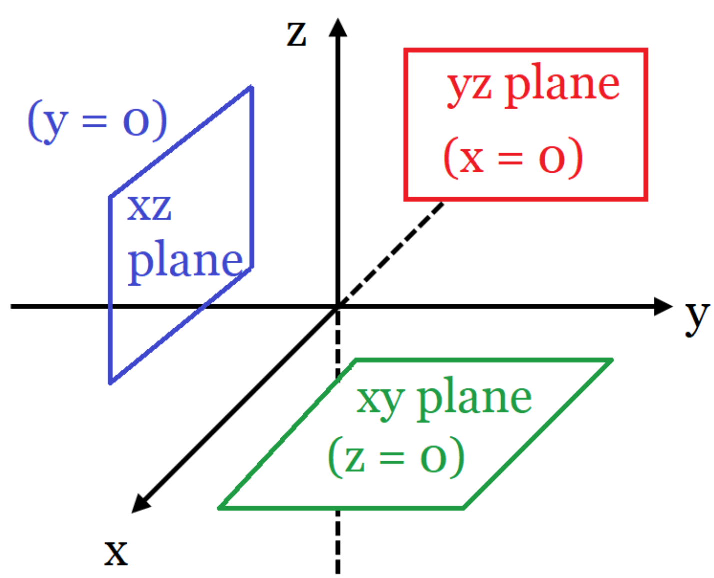
Example 5:

Sketch the region $2 \leq y \leq 3$

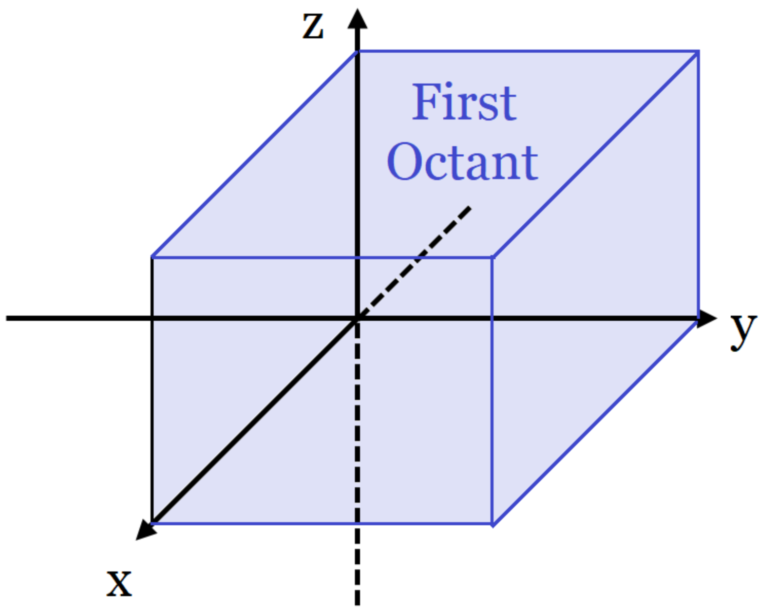
Start by sketching the surfaces $y = 2$ and $y = 3$, which are planes that cross the $y-$axis at 2 and 3, and then $2 \leq y \leq 3$ is everything in between. It kind of looks like an ice cream sandwich, or an electrode in physics.



Related to this are the three most important planes you'll encounter:



And last but not least there is the first **octant**, which is the region where x, y, z are positive and which is the analog of the first quadrant (but in 3 dimensions)



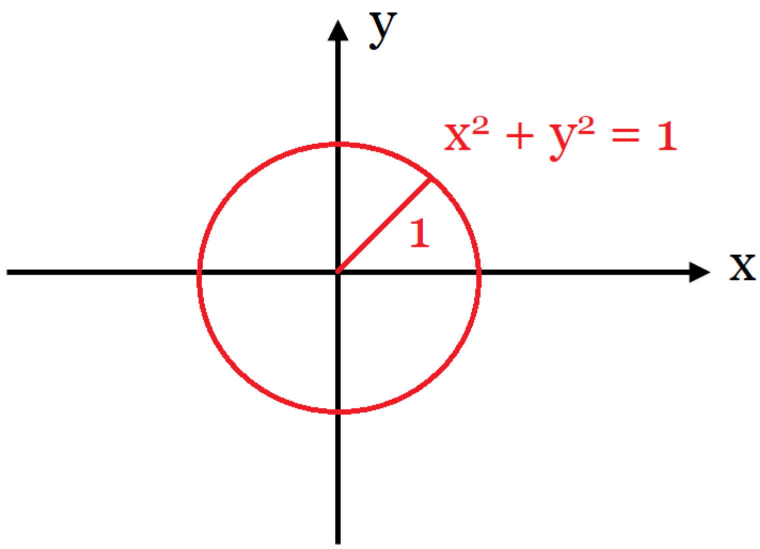
4. Cylinders

Let's discuss an important class of surfaces called cylinders

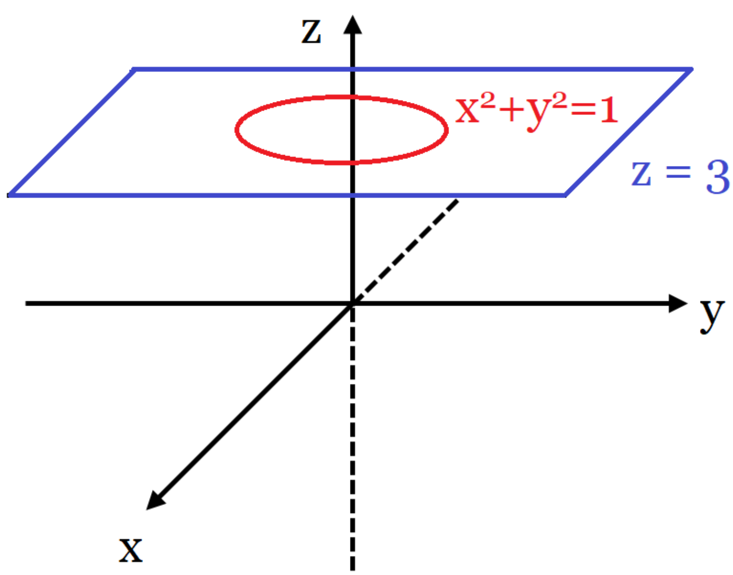
Example 6: (Warm-up)

Sketch the curve $x^2 + y^2 = 1, z = 3$

Note: In two dimensions, $x^2 + y^2 = 1$ is a circle of radius 1



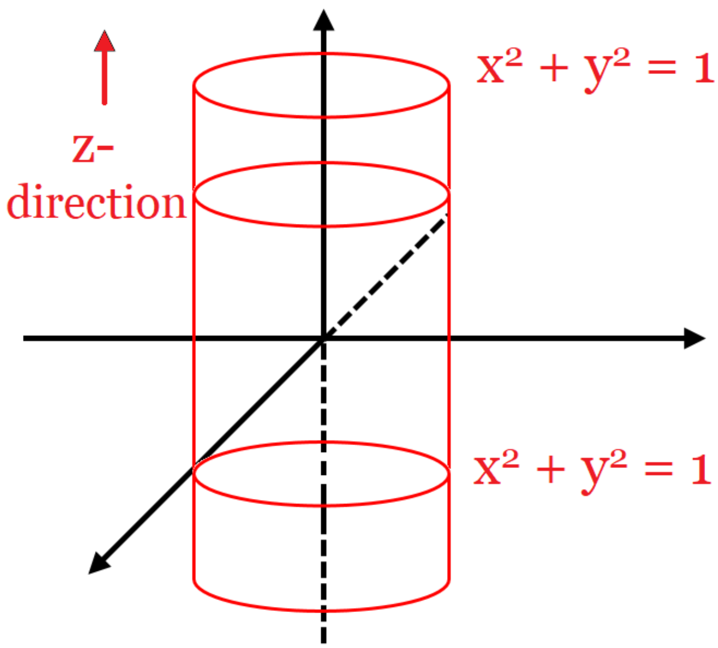
Here it's the same thing, namely it's a circle of radius 1, but with 'height' 3:



Example 7:

Sketch $x^2 + y^2 = 1$

Notice: Here the $z-$variable is missing, so you take the circle $x^2+y^2 = 1$ and you **shift** it along the $z-$direction (which is the missing variable)



And what you obtain in the end is a **cylinder**

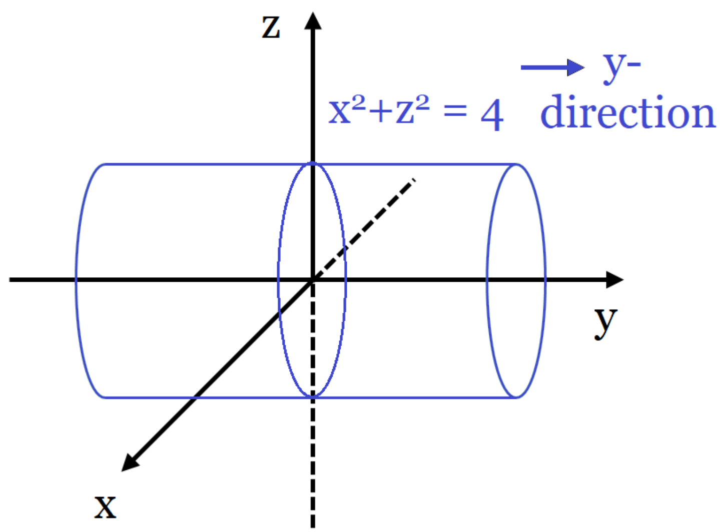
Rule of Thumb

If a variable is missing, it's a cylinder

Example 8:

Sketch $x^2 + z^2 = 4$

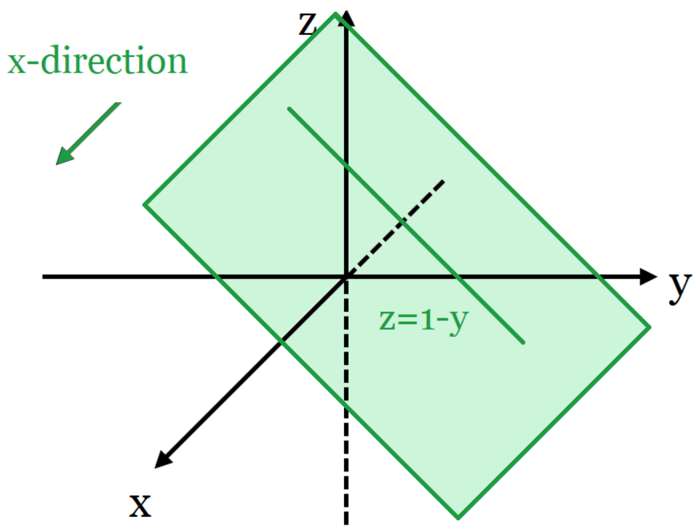
Here y is missing, so you take $x^2 + z^2 = 4$, which is a circle in the $xz-$plane, and shift it in the y-direction



Example 9:

Sketch $y + z = 1 \Rightarrow z = 1 - y$

In the $yz-$plane, this becomes a line. Since x is missing, we need to shift that line in the $x-$direction, so $y + z = 1$ becomes a plane

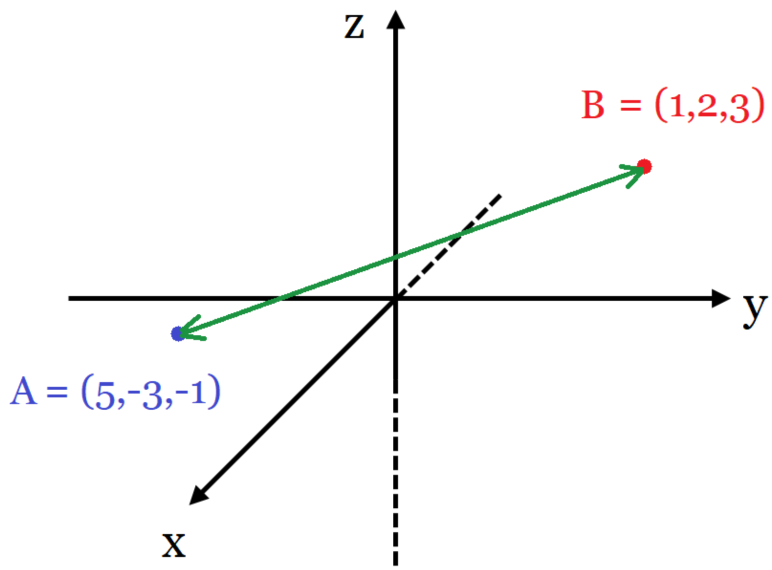


5. Spheres

Finally, let's discuss perhaps the most important surface in this course, the sphere!

Example 10:

Find the distance between $A = (5, -3, -1)$ and $B = (1, 2, 3)$



It's almost identical to the 2-dimensional formula:

$$\begin{aligned}\text{Distance} &= \sqrt{(1-5)^2 + (2-(-3))^2 + (3-(-1))^2} \\ &= \sqrt{4^2 + 5^2 + 4^2} \\ &= \sqrt{16 + 25 + 16} \\ &= \sqrt{57}\end{aligned}$$

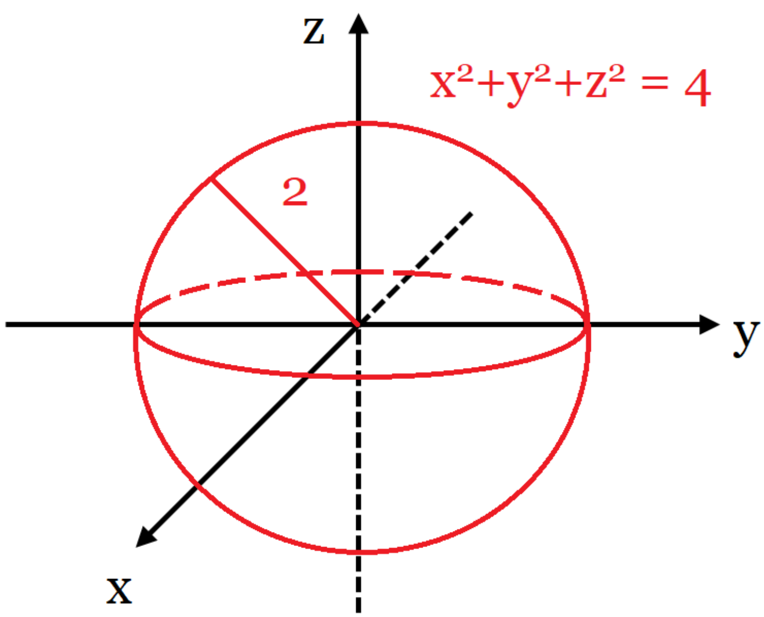
Example 11:

Sketch the surface $x^2 + y^2 + z^2 = 4$

Notice this is equivalent to

$$\begin{aligned}\sqrt{x^2 + y^2 + z^2} &= \sqrt{4} \\ \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} &= 2\end{aligned}$$

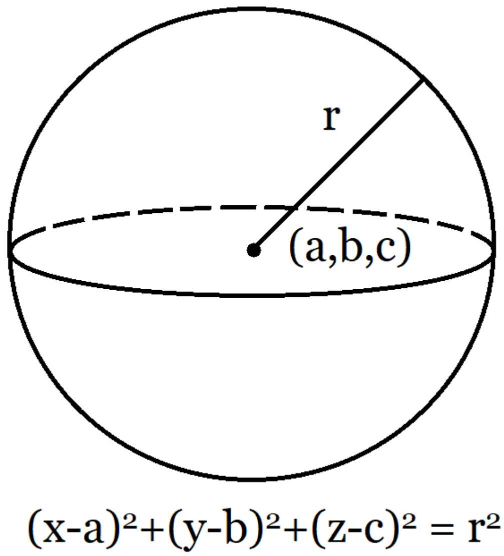
So we're really asking ourselves: Which points are a distance 2 away from $(0,0,0)$? The answer is precisely a sphere centered at the origin $(0,0,0)$ and radius 2.



Fact:

The sphere centered at (a,b,c) and radius r has equation

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$



Why? It has to do with distances, you're just asking yourselves: Which points are a distance r away from (a, b, c)?

Example 12:

The equation of the sphere centered at $(0, 0, 0)$ and radius 3 is $x^2 + y^2 + z^2 = 9$

Example 13:

What is the equation of the sphere centered at $(1, -2, -3)$ and radius 4?

$$\begin{aligned}(x-1)^2 + (y-(-2))^2 + (z-(-3))^2 =& 4^2 \\ (x-1)^2 + (y+2)^2 + (z+3)^2 =& 16\end{aligned}$$

Example 14: (Good quiz/exam question)

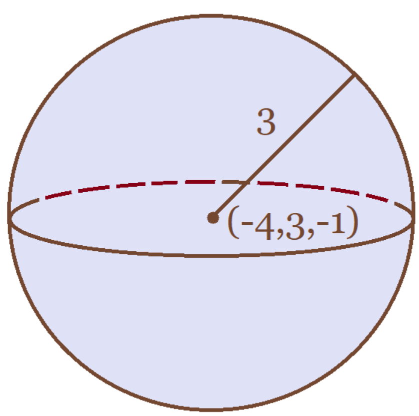
Sketch the region

$$x^2 + y^2 + z^2 \leq -8x + 6y - 2z - 17$$

The trick here is to put everything (except for the -17) on one side and complete the square:

$$\begin{aligned}
x^2 + y^2 + z^2 + 8x - 6y + 2z &\leq -17 \\
(x^2 + 8x) + (y^2 - 6y) + (z^2 + 2z) &\leq -17 \\
(x+4)^2 - 16 + (y-3)^2 - 9 + (z+1)^2 - 1 &\leq -17 \\
(x+4)^2 + (y-3)^2 + (z+1)^2 &\leq -17 + 16 + 9 + 1 \\
(x+4)^2 + (y-3)^2 + (z+1)^2 &\leq 9 \\
(x-(-4))^2 + (y-(3))^2 + (z-(-1))^2 &\leq 9
\end{aligned}$$

This is the inside of the sphere centered at $(-4, 3, -1)$ and radius 3, also called the **closed ball** centered at $(-4, 3, -1)$ and radius 3:



$(x+4)^2+(y-3)^2+(z+1)^2 \leq 9$

Example 15:

Sketch the region

$$1 \leq x^2 + y^2 + z^2 \leq 4, \text{ in the first octant}$$

First of all, $x^2 + y^2 + z^2 = 1$ is the sphere centered at $(0,0,0)$ and radius 1 (unit sphere), and $x^2 + y^2 + z^2 = 4$ is the sphere centered at $(0,0,0)$ and radius 2, so $1 \leq x^2 + y^2 + z^2 \leq 4$ is the "shell" in between.

Finally, remember that the first octant is the region where x, y, z are positive.