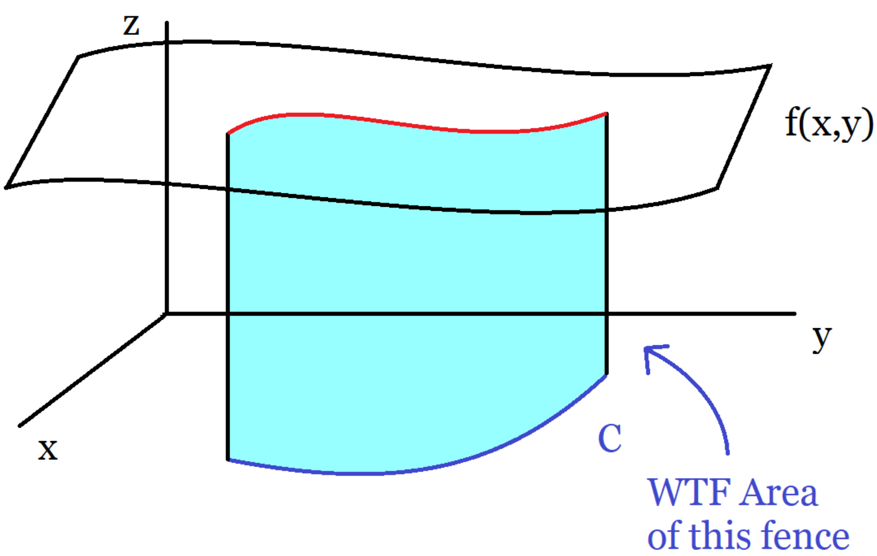
LINE VS SURFACE INTEGRALS

1. For Functions

1.1. **Line Integral of a Function.**

Definition:

$$\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2 + (y'(t))^2}dt$$



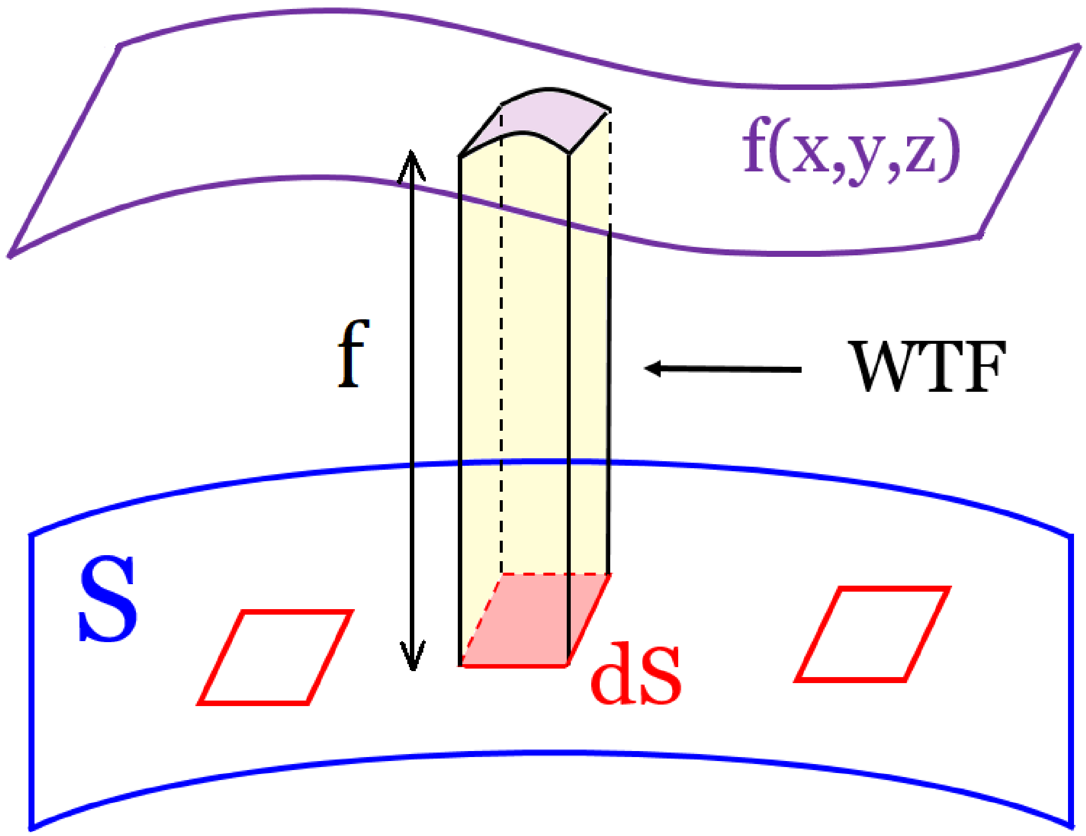
Calculates the area of the fence under f and over C

Date: Thursday, December 9, 2021.

1.2. **Surface Integral of a Function.**

Definition:

$$\int\int_S f(x,y,z)dS = \int\int_D f(r(u,v))\|r_u \times r_v\|\, dudv$$



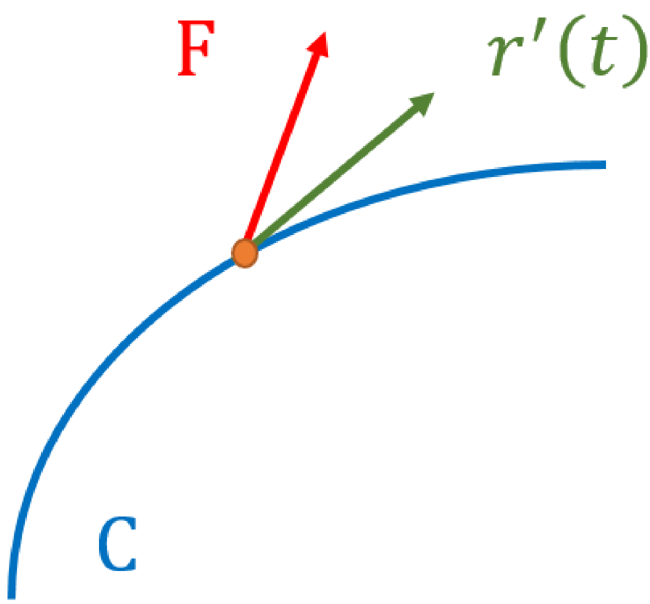
Calculates the volume of the region under f and over S

2. For Vector Fields

2.1. Line Integral of a Vector Field.

Definition:

$$\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)dt$$

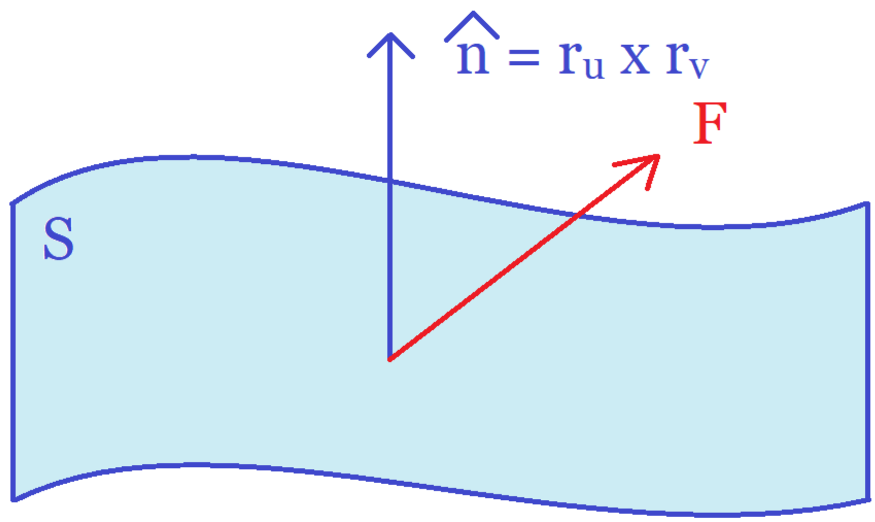


Calculates the work done of F over C. Here you dot F with the **direction** vector of C

2.2. **Surface Integral of a Vector Field.**

Definition:

$$\int\int_S F \cdot d\mathbf{S} = \int\int F \cdot \hat{n} = \int\int_D F(r(u,v)) \cdot (r_u \times r_v) du dv$$



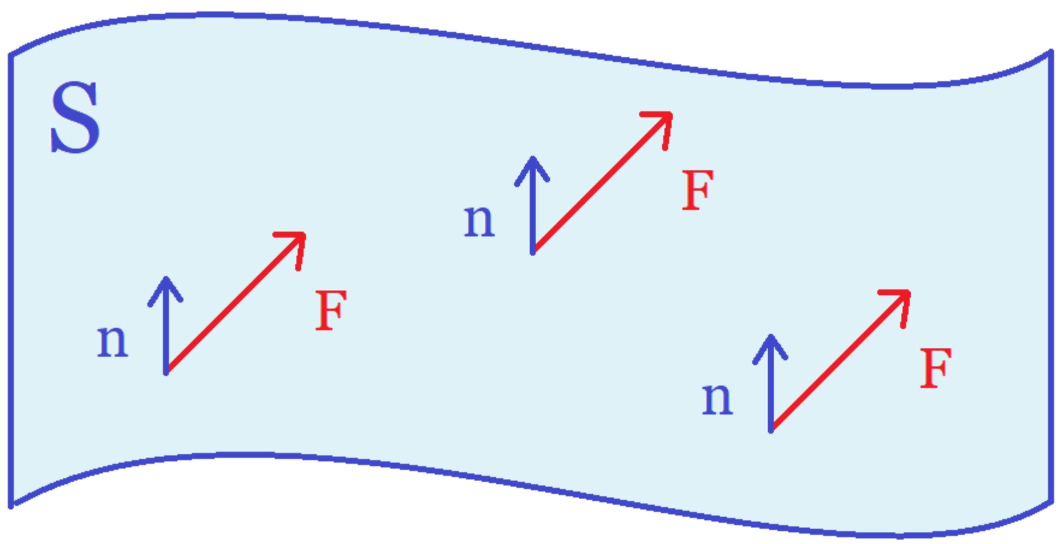
Calculates the net flux of F over S. Here you dot F with the **normal** vector of S

2.3. **"Adult" Version.**

Fact:

$$\int\int_S F \cdot d\mathbf{S} = \int\int_S F \cdot n \, dS$$

Here $n = \frac{\hat{n}}{\|\hat{n}\|}$ is the *unit* normal vector.



This formula relates surface integrals of vector fields with surface integrals of functions, useful for theoretical purposes.

Before you summed $F \cdot \hat{n}$ over D and now you sum $F \cdot n$ over S