MATH 2D – MAKE–UP FINAL

Name: ______________________

Student ID: ______________________

Instructions: This is it, your final hurdle to freedom!!! You have 120 minutes to take this exam, for a total of 100 points. This is a closed book and closed notes exam and calculators and/or portable electronic devices are not allowed. Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. May the Chen Lou be with you!!!

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating will be subject to disciplinary consequences, pursuant to section 102.1 of the UCI Student Code of Conduct.

Signature: ______________________________

1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

Date: Tuesday, June 12, 2018.

1. *(10 points)* Estimate the error in calculating the volume of the box if the box has dimensions $x = 10$, $y = 20$, $z = 30$, and the errors in calculating the sides are $dx = -0.1, dy = 0.2, dz = 0.3$.

2. *(10 points)* Show that $u(x, y) = \ln(x^2 + y^2)$ solves Laplace's equation

$$u_{xx} + u_{yy} = 0$$

3. *(10 points)* The two sub-parts of this problem are independent of each other
   (a) *(5 points)* Suppose that for all t, $\mathbf{u}(t)$ is a unit vector. Show that $\mathbf{u}'(t)$ is always perpendicular for $\mathbf{u}(t)$.

   Hint: $\|\mathbf{u}(t)\|^2 = \mathbf{u}(t) \cdot \mathbf{u}(t)$.

   (b) *(5 points)* Suppose $\mathbf{u}$ is a unit vector. Find the (vector) projection of ∇f on $\mathbf{u}$ and express your answer in terms of $D_{\mathbf{u}} f$ (the directional derivative of f in the direction $\mathbf{u}$). Simplify your answer as much as possible.

4. *(10 points)* Find the equation of the tangent plane to the surface $x^4 + y^4 + z^4 = 3x^2y^2z^2$ at the point $(1, 1, 1)$. Write your answer in the form $ax + by + z = c$ for some a, b, c.

5. *(10 points)* Use **Lagrange multipliers** to find the smallest and largest distance between the point $(1, 2)$ and the circle $x^2 + y^2 = 5$.

6. *(10 points)*

   (a) *(8 points)* Use **Lagrange multipliers** to show that the triangle with maximum area that has a given perimeter $p > 0$ must be equilateral.

   Hint: The area of a triangle with sides x, y, z is $A = \sqrt{s(s-x)(s-y)(s-z)}$ where $s = \frac{p}{2}$. Assume that $s \neq x, y, z$.

   (b) *(2 points)* What kind of triangle do you get if $s = x$?

7. *(10 points)* Find the average value of the function $f(x, y) = \frac{1}{\sqrt{x^2+y^2}}$ on the ring $a^2 \leq x^2 + y^2 \leq b^2$ where $b > a > 0$. Simplify your answer as much as possible

8. *(10 points)* Write $\int_0^2 \int_0^{4-x^2} \int_0^{1-\frac{z}{4}} f(x,y,z)dydzdx$ as

$$\int_?^? \int_?^? \int_?^? f(x,y,z)dxdzdy$$

Illustrate with pictures.

9. *(10 points)* Find the volume of the solid enclosed by the cylinder $x^2 + z^2 = 4$ and the planes $y = -1$ and $y + z = 4$

10. *(10 points)* The Grand Finale!!! Assume $b > a > 0$. Calculate the integral

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$$

Hint: Calculate the volume under the function $z = e^{-xy}$ and over the rectangle $[0, \infty) \times [a, b]$ in two different ways.