MATH 409 – MIDTERM 1

Name	
Student ID	
Signature	

Instructions: Welcome to your Midterm! You have 75 minutes to take this exam, for a total of 50 points. No books, notes, calculators, or cellphones are allowed. **Please write in complete sentences if you can.** Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, please clearly indicate so, or else your work may be discarded.

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Aggie Honor Code.

Date: Thursday, September 30, 2021.

1. *(10 points)* Prove **one** of the following statements (**NOT** both)

   ☐ The Monotone Sequence Theorem (don't state it)

   ☐ If (s_n) converges to s, then (s_n) is bounded

2. *(10 = 2 + 8 points)*

(a) Define: (s_n) converges to s

(b) Use the **definition** of a limit to show that

$$\lim_{n\to\infty} \frac{2n+3}{5n+7} = \frac{2}{5}$$

3. *(10 = 2 + 8 points)*

(a) State the Monotone Sequence Theorem

(b) Define the following sequence (s_n) by $s_1 = 2$ and

$$s_{n+1} = \frac{s_n}{2} + \frac{1}{s_n}$$

Show that (s_n) converges and find its limit.

Hint: First show by induction that $s_n \geq \sqrt{2}$

4. *(10 = 2 + 6 + 2 points)* Let S be a nonempty bounded subset of $\mathbb{R}$

   (a) Define: $\sup(S) = M$ (use the definition from lecture)

   (b) If $k > 0$, then we define $kS = \{ks \mid s \in S\}$

   $$\text{Show that: } \sup(kS) = k \sup(S)$$

   Hint: Let $M = \sup(S)$ and show $\sup(kS) = kM$

   (c) Is (b) true if $k < 0$? Why or why not?

5. *(10 = 2 + 6 + 2 points)* Let (s_n) and (t_n) be two bounded sequences

(a) Define: $\limsup_{n\to\infty} s_n$

(b) Show that:

$$\limsup_{n\to\infty}(s_n + t_n) \leq \left(\limsup_{n\to\infty} s_n\right) + \left(\limsup_{n\to\infty} t_n\right)$$

Hint: Show that this is true for our helper sequence (v_N)

(c) Find two bounded sequences (s_n) and (t_n) such that

$$\limsup_{n\to\infty}(s_n + t_n) \neq \left(\limsup_{n\to\infty} s_n\right) + \left(\limsup_{n\to\infty} t_n\right)$$