MAT 267, Spring 2021, Test 1

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Part I: Multiple Choice. Select the correct answer for each problem and make a table of your answers at the top of the first page of your exam. There are 8 problems, each worth 7 points, for a total of 56 possible points.

1. Which vector is parallel to $\mathbf{u} = \langle 2, 2, -1 \rangle$ and has a magnitude of 2?

A. $\frac{2}{3}\langle 2, 2, -1 \rangle$

B. $\frac{1}{9}\langle 2, 2, -1 \rangle$

C. $\frac{1}{3}\langle 2, 2, -1 \rangle$

D. $\frac{1}{3}\langle -2, -2, 1 \rangle$

E. None of the above

2. The angle between the vectors $\mathbf{u} = \langle 3, 1 \rangle$ and $\mathbf{v} = \langle 1, 2 \rangle$ is

A. $30°$

B. $45°$

C. $60°$

D. $90°$

E. None of the above

3. If $\mathbf{u}$ and $\mathbf{v}$ are parallel vectors in $\mathbb{R}^3$, then $\mathbf{u} \times \mathbf{v}$ is

A. parallel to $\mathbf{u}$ and $\mathbf{v}$

B. the zero vector

C. undefined

D. $|\mathbf{u}||\mathbf{v}|$

E. None of the above

4. Which of the following is a valid parameterization for the intersection curve of the surfaces $x^2 + y^2 = 16$ and $z = 2y$?

A. $\langle 4\cos t, 4\sin t, 2t \rangle$

B. $\langle \cos t, \sin t, 16\sin t \rangle$

C. $\langle 4\cos t, 4\sin t, 8\sin t \rangle$

D. $\langle 16\cos t, 16\sin t, 32t \rangle$

E. None of the above

5. An equation for the line through the points $(2, 4, -5)$ and $(1, 2, 1)$ is

A. $(x-2) + 2(y-4) + (z+5) = 0$

B. $\mathbf{r}(t) = \langle 2t, 4t, -5t \rangle$

C. $\mathbf{r}(t) = \langle -1 + 2t, -2 + 4t, 6 - 5t \rangle$

D. $\mathbf{r}(t) = \langle 2 - t, 4 - 2t, -5 + 6t \rangle$

E. None of the above

6. The tangent vector of $\mathbf{r}(t) = \langle 2t, 3t^2 + 4t, \frac{1}{t} \rangle$ at $t = 1$ is

A. $\langle 2, 10, -1 \rangle$

B. $\langle 2, 7, 1 \rangle$

C. $\langle 1, 3, 0 \rangle$

D. $\langle 1, 5, -1 \rangle$

E. None of the above

7. Let $\mathbf{u} = \langle 2, 1, 1 \rangle$ and $\mathbf{v} = \langle 1, 0, 2 \rangle$. The area of the parallelogram formed by $\mathbf{u}$ and $\mathbf{v}$ is

A. 14

B. $\sqrt{14}/2$

C. 4

D. $\sqrt{14}$

E. None of the above

8. If an object has a position vector $\mathbf{r}(t) = \langle \cos(2t), 3t^2 + t^3, e^{-3t} \rangle$, then its acceleration vector at $t = 0$ is

A. $\langle 4, 6, \frac{1}{9} \rangle$

B. $\langle 1, 0, 1 \rangle$

C. $\langle -4, 6, 9 \rangle$

D. $\langle -\frac{1}{4}, 0, \frac{1}{9} \rangle$

E. None of the above

Part II: Free Response. Solve each problem and show all work clearly. Draw a box around your final answers. There are 3 problems worth a total of 44 points.

1. [14 points] Consider the curve $\mathbf{r}(t) = \langle \cos(2t), \sin(2t), 4t + 3 \rangle,\ 0 \le t \le 2\pi$.

a) [7 pts] Find the arc length of the curve. Give an exact symbolic answer, no approximations.

b) [7 pts] Find a vector equation of the tangent line $\mathbf{L}(t)$ to the curve when $t = \pi/2$.

2. [14 points] A plane in $\mathbb{R}^3$ contains the points $(2, 1, 1)$, $(1, 4, -1)$ and $(3, 2, 0)$.

a) [7 pts] Find the values of b and c such that $\mathbf{n} = \langle 1, b, c \rangle$ is a normal vector for the plane.

b) [7 pts] Find the equation of the plane and state it in the form $Ax + By + Cz = D$.

3. [16 points] A particle has a velocity vector $\mathbf{v}(t) = \langle 2t - 3, 4\sin t, 6e^{-2t} \rangle$ m/s and an initial position $\mathbf{r}(0) = \langle 2, 4, 3 \rangle$ m.

a) [8 pts] Find the position function $\mathbf{r}(t)$ of the particle.

b) [8 pts] If the particle has a mass of 5 kg, find the magnitude of the force acting on it when $t = 0$ (Exact answer please)