MATH 251 – MIDTERM 1 – STUDY GUIDE

The Midterm takes place on **Friday, September 24, 2021** during the usual lecture time and in the regular lecture room. It will be an in-person exam and **NO** books/notes/calculators/cheat sheets will be allowed. Please bring your student ID card (or other government ID), for verification purposes. The midterm counts for 20 % of your grade, and covers Chapters 12 and 13.

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. For a more thorough study experience, look at all the lectures, quizzes, practice exams, and the suggested homework.

Format: There are *tentatively* 5-6 questions on the exam, all of them free response, no multiple choice. The reason everything is free response is because in that way I can give you partial credit.

Section 12.1: Three-Dimensional Coordinate Systems

- Plot points in 3 dimensions, like $(5, 3, -1)$.
- QUICKLY sketch surfaces like $z = 3$, $x = 3$, $y = 3$ but also regions like $-1 \leq y \leq 2$
- QUICKLY sketch cylinders like $x^2 + y^2 = 1$ or $x^2 + z^2 = 4$
- Know the equation of a sphere
- Know how to find equations by completing the square, like the one on the quiz. I could ask the same question for the surfaces in section 12.6 as well
- Draw regions like $1 \leq x^2 + y^2 + z^2 \leq 4$

Section 12.2: Vectors

- Hopefully this section is not too crazy for you!
- Know what it means for vectors to be parallel
- Find the length (magnitude) of a vector
- Produce a unit vector in the same direction as a given vector
- Ignore the section "Crazy Physics Problem" in the (full) lecture notes

Section 12.3: The Dot Product

- Know the definition of the dot product, the Angle Formula, and the fact that two vectors are perpendicular if and only if their dot product is 0.
- Find the angle between vectors
- Find the work done of a force on a displacement

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- Know the formula for $\hat{\mathbf{b}} = \text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\frac{\mathbf{b}\cdot\mathbf{a}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a}$, the vector projection of $\mathbf{b}$ on a vector $\mathbf{a}$, as well as the equation for scalar projection $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|}$ (the length of the vector projection, but without the absolute value). The way to remember this is: First you want a multiple of $\mathbf{a}$, so $\hat{\mathbf{b}} = c\mathbf{a}$ and to find c, use the hugging analogy in lecture!
- Ignore the section in the book on direction angles and direction cosines

Section 12.4: The Cross Product

- Know the definition of the cross product of two vectors
- The most important property about cross products you need to know is that $\mathbf{u} \times \mathbf{v}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$
- Find the area of a parallelogram, both in 2 dimensions and in 3 dimensions. This also includes the area of a triangle, which is just half a parallelogram
- Find the volume of a parallellipiped
- Know the angle formula $\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\,\|\mathbf{b}\|\sin(\theta)$
- Ignore the sections on Triple Products
- Find the torque of a force $\mathbf{F}$ on a position $\mathbf{r}$

Section 12.5: Lines and Planes

- This is definitely the trickiest section for the midterm, because there are so many things that I could ask you! That said, please remember the following:
  - To find the equation of a line, find a **point** on the line and a **direction vector**
  - To find the equation of a plane, find a **point** on the plane and a **normal vector**
- Here are some line questions I could ask you: (all of those can be found in the lecture notes)
  - Find the equations of a line through a point and with a given direction vector
  - Find equations for a line through two points
  - Where does a line with a given parametric equation intersect a given plane?
  - Find equations of a line parallel to another one
  - Find the symmetric equations of a line
  - Find the equation of a line segment
  - Are two lines (with given equations) parallel, intersecting, or skew? If so, find the point of intersection
- Here are some plane questions I could ask you: (all of those can be found in the lecture notes)
  - Find the equation of a plane going through a point and with a given normal vector
  - Find a plane containing three points
  - Find a plane parallel to another one
  - Find a plane containing a point and a line
  - Find a plane containing two lines

  - ▶ Find the angle between two planes
  - ▶ Find a plane perpendicular to a line
  - ▶ Find the line of intersection between two planes.
  - ▶ Find a line perpendicular to a plane
  - ▶ Find a plane equidistant (same distance) to two points
- Find the distance between a point and a plane
- Find the distance between two parallel planes

Section 12.6: Cylinders and Quadric Surfaces

- **YOU ABSOLUTELY NEED TO MEMORIZE THE 8 IMPORTANT SURFACES!!!!!!!** It is crucial for your survival for the rest of the course! I accept the terms "Saddle" (for hyperbolic paraboloid), "Dress" (for hyperboloid of one sheet), and "Two cups" (for hyperboloid of two sheets), and for elliptic paraboloid you can just call it "Paraboloid." Know the names and how to draw them
- You don't have to use traces if you find that useless
- I could ask you a question about completing the square, like on the Quiz question.
- I could ask you those surfaces but in another direction (like $x^2 = y^2 + z^2$, which is a cone in the x direction)

Section 13.1: Vector functions and Space Curves

- Sketch some vector functions (see lecture notes)
- Find parametric equations for the intersection of two surfaces
- Find domains of vector functions
- Skip the section "Using Computers to Draw Space Curves"

Section 13.2: Derivatives and Integrals of Vector Functions

- Find limits of vector functions; no need to worry about continuity
- Find derivatives of vector functions and parametric equations of tangent lines at a point
- Find the unit tangent vector to a curve
- Know the formula $(\mathbf{u} \cdot \mathbf{v})' = \mathbf{u}' \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{v}'$ and similar for cross products
- Show that if $\|\mathbf{r}(t)\| = 3$ then $\mathbf{r}'(t) \cdot \mathbf{r}(t) = 0$
- Find Integrals of vector functions; remember to have different constants!

Section 13.3: Arclength and Curvature

- In this section, I'll only ask about arclength, ignore everything about curvature and binormal vectors

Section 13.4: Motion in Space: Velocity and Acceleration

- Find velocity, acceleration, and speed, given a position vector
- Find position given an acceleration (or velocity) vector
- Know Newton's second law, $\mathbf{F} = m\mathbf{a}$ and use it to find $\mathbf{F}$ and $\|\mathbf{F}\|$
- Ignore the example on projectile motion