1. *(10 points)*

   Let (s_n) be the sequence defined by $s_1 = 2$ and, for all $n \in \mathbb{N}$:

$$s_{n+1} = \sqrt{2 + s_n}$$

   Use induction on n to show that $s_n \leq 2$ for all $n \in \mathbb{N}$

2. *(10 points)*

Use contradiction to prove that for every $x \in \mathbb{R}$ there is an integer $k \in \mathbb{Z}$ such that $k > x$.

Do **not** use the Archimedean property!

Hint: At some point you might consider $M - 1$ (for some M).

3. *(10 points)*

Let f be a function with the following property: There is $C > 0$ such that, for all a and b, $|f(b) - f(a)| \leq C\,|b - a|$

Let (s_n) be a sequence in $\mathbb{R}$ that converges to s. Show that $f(s_n)$ converges to $f(s)$. Include your scratchwork.

Note: $f(s_n)$ means "f of s_n" and $f(s)$ means "f of s"

4. *(10 points)*

Let (s_n) be a sequence in $\mathbb{R}$ that is bounded above by M. Suppose moreover that (s_n) converges to M. Show that

$$\sup \{s_n \mid n \in \mathbb{N}\} = M$$