MATH 409 – MIDTERM 2

Name	
Student ID	
Signature	

Instructions: Welcome to your Midterm! You have 75 minutes to take this exam, for a total of 50 points. No books, notes, calculators, or cellphones are allowed. **Please write in complete sentences if you can.** Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, please clearly indicate so, or else your work may be discarded.

Academic Honesty Statement: With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Aggie Honor Code.

Date: Thursday, November 4, 2021.

1. *(12 points)* Prove **<u>one</u>** of the following statements (**NOT** both)

☐ If (s_n) is a sequence that converges to $s > 0$ and (t_n) is any bounded sequence, then

$$\limsup_{n\to\infty} s_n t_n = \left(\limsup_{n\to\infty} s_n\right)\left(\limsup_{n\to\infty} t_n\right)$$

☐ If $f : [a, b] \to \mathbb{R}$ is continuous, then f is bounded, that is: There is M such that $|f(x)| \leq M$ for all x

2. *(12 = 2 + 10 points)*

(a) Define: f is continuous at x_0 (the $\epsilon - \delta$ definition)

(b) Use $\epsilon - \delta$ to show that $f(x) = 3x^2 - 5$ is continuous at x_0

3. *(14 = 2 + 8 + 2 + 2 points)*

(a) State the Cauchy Criterion for convergence of a series $\sum a_n$

(b) Suppose (a_n) is a sequence of positive terms such that $\sum_{n=1}^{\infty} a_n$ converges. Use (a) to show that $\sum_{n=1}^{\infty} (a_n)^2$ converges

Hint: What happens to a_n as $n \to \infty$?

(c) Find a sequence (a_n) such that $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} (a_n)^2$ diverges. *Briefly* justify your answer

(d) Find a sequence (a_n) such that $\sum_{n=1}^{\infty} (a_n)^2$ converges, but $\sum_{n=1}^{\infty} a_n$ diverges. *Briefly* justify your answer.

4. *(12 points)*

Let S be a nonempty bounded subset of $\mathbb{R}$ such that $\inf(S) \notin S$

Use an **inductive construction** to find a decreasing sequence (s_n) in S such that $s_n \to \inf(S)$

Do **not** use sup

(Scratch paper)