MAT 267, Spring 2021, Test 2

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Part I: Multiple Choice. Select the correct answer for each problem. Make a table of all your answers at the top of the first page of your exam. There are 8 problems, each worth 7 points, for a total of 56 possible points.

1. The domain of the function $f(x,y) = \ln(x) + \ln(y)$ is

A. the entire xy plane

B. the first and second quadrants of the xy plane

C. the first quadrant of the xy plane

D. the first and third quadrants of the xy plane

E. none of the above

2. The level curve $z = 1$ of the function $z = 6x^2 + 3y^2$ is a

A. parabola

B. ellipse

C. straight line

D. hyperbola

E. none of the above

3. The equation of the tangent plane of the surface $z = 3x^2 - \frac{1}{y}$ at the point $(2,-1)$ is

A. $z = 12x + y - 23$

B. $z = 12x + y + 13$

C. $z = 12x + y - 12$

D. $z = 12x + y - 10$

E. none of the above

4. If $w = 2xy - 3y$, $x = 3u - 2v$, and $y = v^2 \sin u$, then $\partial w/\partial u$ equals

A. $(2y)(3) + (2x-3)(2v\sin u)$

B. $(2y)(-2) + (2x-3)(2v\sin u)$

C. $(2y)(3) + (2x-3)(v^2\cos u)$

D. $(2y)(-2) + (2x)(v^2\cos u)$

E. none of the above

5. Which of the following is a normal vector for the surface $x^2yz^4 - 2z = 1$ at the point $(1,3,1)$?

A. $\langle 6,1,10\rangle$

B. $\langle -1,-3,-1\rangle$

C. $\langle 6,1,12\rangle$

D. $\langle 1,6,4\rangle$

E. none of the above

6. Which of the following represents the double integral $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \cos(x^2+y^2)\, dy\, dx$ in polar coordinates?

A. $\int_0^{\pi}\int_0^3 \cos(r^2)\, dr\, d\theta$

B. $\int_0^{\pi}\int_0^3 r\cos(r^2)\, dr\, d\theta$

C. $\int_0^{\pi}\int_0^3 r\cos(r)\, dr\, d\theta$

D. $\int_0^{2\pi}\int_0^3 r\cos(r^2)\, dr\, d\theta$

E. none of the above

7. Calculate $\iint_D (x^2+y^2)^{5/2}\, dx\, dy$ where D is the disk of radius 1 centered at $(0,0)$ in the first quadrant

A. $\frac{\pi}{5}$

B. $\frac{\pi}{7}$

C. $\frac{\pi}{10}$

D. $\frac{\pi}{14}$

E. none of the above

8. If D is the region bounded by the lines $y = 0$, $x = 2$ and $y = 2x$, then the value of the double integral $\iint_D 1\, dy\, dx$ is

A. 1

B. 2

C. 4

D. 8

E. none of the above

Part II: Free Response. Solve each problem and show all work clearly. Draw a box around your final answers and include units where applicable. Give exact answers to all questions; do NOT state the answers as decimal approximations There are 3 problems worth a total of 44 points.

1. [16 points] Consider the function $T(x, y, z) = x^2y + y^2z - 3xz$.

a) [8 pts] Evaluate the gradient vector of T at the point $P = (1, 2, -1)$.
b) [4 pts] Find the maximal rate of increase of T at the point P.
c) [4 pts] Find the directional derivative of T at the point P in the direction of $\mathbf{v} = \langle 1, 1, 1 \rangle$.

2. [14 points] Consider the function $f(x, y) = x^3 + y^2 - 6xy$.

a) [7 pts] Find all critical points of f.
b) [7 pts] Classify all critical points as either a local maximum, local minimum or saddle point.

3. [14 points] Consider the following *impossible* integral

$$\int_0^1 \int_{x^2}^1 \frac{xy}{1+y^3}\, dy\, dx.$$

a) [4 pts] Draw a clear picture of D (the region of integration in the xy plane), labeling all functions
b) [4 pts] Re-write the integral in terms of $dx\ dy$ instead of $dy\ dx$.
c) [6 pts] Evaluate the integral from part (b).