MATH 251 – MIDTERM 2 – STUDY GUIDE

The Midterm takes place on **Friday, October 15, 2021** during the usual lecture time and in the regular lecture room. It will be an in-person exam and **NO** books/notes/calculators/cheat sheets will be allowed. Please bring your student ID card (or other government ID), for verification purposes. The midterm counts for 20 % of your grade, and covers all of Chapter 14 (except section 14.2)

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. For a more thorough study experience, look at all the lectures, quizzes, practice exams, and Webassign.

Format: Just like last time, there are *tentatively* 5-6 questions on the exam, all of them free response, no multiple choice.

Note: For this midterm, you do **NOT** need to know the 8 surfaces, but you'll need it for the other exams.

Section 14.1: Functions of Several Variables

- Find the domain of a function of two or three variables, check out the lecture notes for good examples.
- You don't need to worry about sketching the graph of a function, and I won't ask you about range or limits

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- Know what a level curve/surface is. I could ask you to sketch specific level curves/surfaces of a function of two/three variables, like given $z = x^2 + y^2$, plot $z = 1$ or $z = 2$
- Check out Examples 9 and 10 in the Lecture 9 notes, aren't they neat?
- You don't need to know how to plot a function given a level curve

Note: Ignore Section 14.2: Limits and Continuity

Section 14.3: Partial Derivatives

- Calculate partial derivatves f_x and f_y (and f_z) of a function, either in general or at a point.
- You also need to know how to do that with implicit equations, like Example 6 in the Lecture 10 notes, but remember that there is another method of doing this in section 14.5
- You don't need to know the definition of partial derivatives as a limit, but just know that they represent slopes in the $x-$direction and the $y-$direction (see the picture on page 3 of the Lecture 10 notes)
- Calculate higher-order partial derivatives like f_{xx} or f_{xy} or $f_{xxxxyyyyxxx}$ (hopefully not that complicated ☺)
- Know Clairaut's theorem: $f_{xy} = f_{yx}$
- The only PDE thing I can ask you is to check that a given function solves a differential equation, like Example 7 in the Lecture 10 notes

Section 14.4: Tangent Planes and Linear Approximations

- Find the equation of a tangent plane of a function at a point

- Find the linear approximation of a function at a point and use it to approximate a certain number, like $\sqrt{(4.01)^2 + (2.99)^2}$. Sometimes you would need to guess the function

- Use differentials to approximate a certain number, like $\ln(0.97)e^{0.02}$ in the Lecture 13 notes

- Use differentials to approximate a certain error, like the last example in the Lecture 13 notes.

- You would need to know volumes and surface areas of basic surfaces, like a box, cylinder, sphere, or a cone (for the cone, just the volume, which is $\frac{1}{3}\pi r^2 h$)

Section 14.5: The Chain Rule

- Make sure you are comfortable with the Chen Lu; remember that the diagram is the key

- Use the chain rule to find $\frac{\partial z}{\partial t}$, where $z = z(x, y)$ and x and y are functions of t I may ask you to do this at specific points (say $t = 0$).

- Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$, where $z = z(x, y)$ and x and y are functions of s and t (or more variables).

- Of course all this works for functions of any variables, like x, y, z

- Find derivatives of implicit functions, like $\frac{\partial z}{\partial x}$ where $x^3 + y^3 + z^3 + 6xyz = 1$. Here it's up to you: You can either do it directly, using the methods in section 14.3, or you can use the formula in the lecture notes, whichever you find easier

- I could phrase all that in terms of real-life problems, like the problems on Webassign

- I will **not** ask about higher-order partial derivatives, like f_{ts} (see last example in the lecture notes)

Section 14.6: The directional derivative and the gradient vector

- Remember that to calculate directional derivatives, you need to have a **unit** vector!

- Find the gradient ∇f of a function, and evaluate it at a point P.

- **Know that the ∇f is perpendicular to level curves (or surfaces) of f.**

- Find the directional derivative $D_{\mathbf{u}} f$, where $\mathbf{u}$ is given.

- If I just give you an angle θ, this means you have to use $\mathbf{u} = \langle \cos(\theta), \sin(\theta) \rangle$

- Find in which direction f increases the fastest/slowest, and find the maximal/minimal rate of increase of f. You don't need to know the proofs

- Find the equation of the tangent plane or normal line to a given surface at a given point.

- Also make sure to look at problems 51, 52, and 61 in section 14.6. I particularly like this one: Sum of intercepts

Section 14.7: Maximum and Minimum Values

- What makes this section hard is all the cases you have to do for the critical points. Also it's very easy to make algebra mistakes here, so make sure to practice with the algebra. All the Webassign problems are great practice.

- Here's my suggestion: First find the critical points, then apply the saddle point test. If the determinant is negative (sad), then it's a **sad**dle. If it's positive, then move on and apply the Second Derivative test and look at f_{xx}. The diagram on page 6 of the Lecture 16 notes summarizes everything quite nicely

- Find and classify the critical points of a function, that is, say if f has a local maximum, local minimum, or a saddle point at a critical point.

- Find the local max/min/saddle points of a function (same thing, but you calculate f at the point)

- Solve word problems using max/min. Some examples include, but are not limited to:
  - ▶ Find the point on the plane (or a surface) that is closest to a given point; remember to square things with square roots! (see notes)
  - ▶ Find the smallest surface area of a box with a given volume (see notes)
  - ▶ Find the biggest volume of a box with a given surface area (similar)

- ▶ Find the largest possible volume of a box where one point is on a function (see webassign)

- **Note:** In word problems, **UNLESS** I tell you not to use the second derivative test, you **HAVE** to use the second derivative test!

- Find the absolute max/min of a function on a triangle (lecture notes) or rectangle (lecture notes) or quarter circle (Webassign). Remember that for **absolute** max/min, you do **NOT** have to use the second derivative test.

Section 14.8: Lagrange multipliers

- Remember that for Lagrange multipliers, you do **NOT** need to use the second derivative test!

- Find the absolute max/min of f with a given constraint g. Here are some examples of tricks to remember:

  - ▶ Do it by cases (see Example 5 in the Lecture 17 notes) For example, if you have $x = \lambda x$, then $x(1-\lambda) = 0$, so $x = 0$ or $\lambda = 1$, and this gives you already 2 cases.
  - ▶ Solve for x, y, z in terms of λ (Example 6 in the Lecture 17 notes or Example 2 in the Lecture 18 notes). For example, if $x - 3 = \lambda x$, then $x(1-\lambda) = 3$, and $x = \frac{3}{1-\lambda}$, and you do the same for y and z, and you plug it into the constraint to solve for λ.
  - ▶ Set everything equal to λ, like the IKEA problem.

- Remember to use the constraint **last**.

- Find the absolute max/min of f on a region, like Example 1 in the Lecture 18 notes. This just means find the critical points and use Lagrange multipliers. Make sure your critical points are in fact inside your region!

- Solve word problems using Lagrange multipliers. Again, some examples include, but are not limited to:

  - ▶ Find the point on the plane (or a surface) that is closest to a given point remember to square things with square roots!
  - ▶ Find the biggest volume of a box with a given surface area (like the IKEA problem)
  - ▶ Find the smallest surface area of a box with a given volume (similar)
  - ▶ There are also some nice problems in the practice exams

- **Note:** Don't think that you can just skip Lagrange multipliers; I may ask you to do a max/min problem where you **HAVE** to use Lagrange multipliers!

- You don't need to worry about Lagrange multipliers with two constraints