1. *(10 points)*

Suppose S is a nonempty subset of $\mathbb{R}$ with $\sup(S) = \infty$ (this means that for all M, there is $s \in S$ with $s > M$)

Use an **inductive construction** to find an increasing sequence (s_n) in S such that $s_n > n$ for all $n \in \mathbb{N}$

2. *(10 points)*

Show **directly** (without using lim sup or the ratio or root tests) that if (a_n) is a sequence of non-negative real numbers with

$$\liminf_{n\to\infty} |a_n|^{\frac{1}{n}} = \alpha > 1$$

Then $\sum a_n = \infty$

Note: Start by letting $\epsilon > 0$ be such that $\alpha - \epsilon > 1$. Here you want the inf to be *bigger* than something.

3. *(10 = 4 + 4 + 2 points)*

Let E be the following subset of $\mathbb{R}$ (with its usual metric):

$$E = \left\{ 2 - \left(\frac{1}{n}\right) \;\middle|\; n \in \mathbb{N} \right\}$$

Find the following, with justification:

(a) The interior E° of E

(b) The closure $\overline{E}$ of E (the book uses E^-)

(c) The boundary ∂E of E

4. *(10 points)*

Let (S, d) be a **compact** metric space (not necessarily in $\mathbb{R}$ or $\mathbb{R}^k$) and let $F_1 \supseteq F_2 \supseteq F_3 \supseteq \dots$ be a non-increasing sequence of nonempty closed sets F_n

Show that the intersection $\bigcap_{n=1}^{\infty} F_n$ is nonempty.

Hint: Suppose $\bigcap_{n=1}^{\infty} F_n = \emptyset$ and consider the open cover

$$\mathcal{U} = \{F_1^c, F_2^c, \dots\}$$

What set does $\mathcal{U}$ cover? (Here F_n^c is the complement of F_n in S. Notice that $F_1^c \subseteq F_2^c \subseteq \dots$)