MATH 251 – MIDTERM 3 – STUDY GUIDE

The Midterm takes place on **Friday, November 12, 2021** during the usual lecture time and in the regular lecture room. It will be an in-person exam and **NO** books/notes/calculators/cheat sheets will be allowed. Please bring your student ID card (or other government ID), for verification purposes. The midterm counts for 20 % of your grade, and covers all of Chapter 15 (except for sections 15.4 and 15.5)

This is the study guide for the exam, and is just meant to be a *guide* to help you study, just so we're on the same place in terms of expectations. For a more thorough study experience, look at all the lectures, quizzes, practice exams, and Webassign.

Format: Just like last time, there are 5 questions on the exam, all free response, no multiple choice. There's definitely a time crunch here, so make sure to be super comfortable with evaluating integrals. And don't spend 10 mins on a simple $u-$substitution!

YouTube Playlist: Double and Triple Integrals

Note: For this midterm, you **NEED** to know the 8 surfaces, so make sure to study them. Also, I might force you to draw pictures, so don't think you can get by without drawing them.

Useful trig identities to know:

(1) $\sin^2(x) + \cos^2(x) = 1$

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(2) $1 + \tan^2(x) = \sec^2(x)$

(3) $\cos(-x) = \cos(x), \sin(-x) = -\sin(x)$

(4) $\sin(2x) = 2\sin(x)\cos(x)$, $\cos(2x) = \cos^2(x) - \sin^2(x)$

(5) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$, $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

Useful integration techniques to know:

(1) $u-$substitution

(2) Integration by parts, but I'll only ask you about easy cases like $\int xe^x dx$

(3) $\int \cos^2(x)dx, \int \sin^2(x)dx$; check out this video in case you forgot

(4) Trig integrals like $\int \cos^3(x)\sin(x)dx$

(5) $\int \cos^3(x)dx$ and $\int \sin^3(x)dx$

(6) **No** trig sub and **No** partial fractions on this exam

Section 15.1: Double Integrals over Rectangles

- I will **not** ask you about Riemann sums, nor will I ask you about the definition of the integral in terms of Riemann sums.
- Calculate a double integral over a rectangle
- Remember that sometimes it's useful (or even necessary) to interchange dx and dy, like $\int_0^\pi \int_1^2 x\cos(xy)dxdy$

- Evaluate a double integral by interpreting it as a volume (like problems 9 or 12 in 15.1 or the problems in lecture about the sphere or the cylinder, see also 66 in 15.2)
- Find the average value of f over a given region (see also 62 in 15.2)
- Remember the Ravioli problem, and in particular the formula that helps us simplify integrals of the form $f(x)g(y)$

Section 15.2: Double Integrals over General Regions

- Find the double integral of a function over a general region. Sometimes the region is vertical, so you'll have to do Smaller $\leq y \leq$ Bigger, and sometimes it's horizontal, in which case you have to solve for x in terms of y and do Left $\leq x \leq$ Right. This video is a good example of it.
- Find the volume of a given solid by using $\int\int$ Bigger $-$ Smaller , although this is easier done with triple integrals, to be honest
- Evaluate an "impossible" integral by changing the order of integration; for example, write $\int_0^1 \int_x^1 \sin(y^2) dy dx$ as $dxdy$ and evaluate the integral, see this video or the quiz question. I would tell you beforehand that the integral is impossible to evaluate
- Find the average value of a function
- You don't really need to memorize the properties at the end of the Double Integral lecture, hopefully they are obvious to you. One useful property is that if $f \geq 0$ then $\int\int f \geq 0$. So if you get a negative answer in this case, you should be worried.

Section 15.3: Double Integrals in Polar Coordinates

- The hardest trig integral I can ask you is $\int \sin^2(x)$ or $\int \cos^2(x)$ or $\int \cos^3(x)$ or $\int \sin^3(x)$. I won't ask you about $\int \sin^4(x)$ or stuff like that.

- Evaluate an integral by changing to polar coordinates. Remember that this is excellent if you see $x^2 + y^2$ or your region is a disk or a wedge or a ring (annulus). **Don't forget about the r!!!** check out this video and this video

- Find the volume between two surfaces; in particular remember the ice cream cone problem I did in lecture

- Calculate $\int_{-\infty}^{\infty} e^{-x^2} dx$ (you can check out this YouTube video). Don't worry about the problem with $\int_{-\infty}^{\infty} \sin(x^2) dx$

- Ignore the problem in the notes with the semicircle centered at $(1, 0)$

Section 15.6: Triple Integrals

- You **don't** need to know the definition of the integral or Fubini's theorem (but of course know how to use it) and you don't need to know the names of the regions. The only applications you need to know are volumes, average values, and mass.

- Find a triple integral over a general 3D region; good examples of regions are either tetrahedra, or regions between two surfaces (like in lecture); *usually* you have to do Smaller $\leq z \leq$ Bigger

- Use triple integrals to find the volume of a solid

- Find the average value of a function over a $3D$ region

- Remember that sometimes you region faces the $y-$ direction (the book calls this type 3), in which case you have to do Left $\leq y \leq$ Right or the $x-$direction (type 2), in which case you have to do Back $\leq x \leq$ Front.

- Sketch the solid whose volume is given by a given integral.

- I will definitely **NOT** ask you to change the order of integration for a triple integral, but I *could* ask you to do that for a double integral.

- Review the problems from lecture, those are very good sample questions, like Gouda Cheese or Cannoli

- Know how to find the volume of intersection of 2 cylinders, like in this video.

- Remember that, while a volume has to be positive, a triple integral could be negative

Section 15.7: Cylindrical Coordinates

- There is absolutely nothing new to learn in this section, it's basically polar coordinates, except you add a z.

- Calculate triple integrals using cylindrical coordinates; problems 17 through 24 in 15.7 are excellent practice problems

- Remember that cylindrical coordinates are useful if you have cylinders or if you see $\sqrt{x^2+y^2}$

- Gotta graph them all, gotta graph them all, Surfaces!!!

- I won't ask you to plot points in cylindrical coordinates, nor will I ask to change a point from cylindrical to rectangular coordinates (and vice-versa)
- All the problems in the lecture notes are fair game for the exam. In particular check out Cylindrical Integral, Integral over Princess Cake and Integral over SIMS

Section 15.8: Spherical Coordinates

- I could ask you to rederive the equations for spherical coordinates, see this video.
- Memorize the equations for spherical coordinates! For x and y, think $x = r\cos(\theta)$ and $y = r\sin(\theta)$ and use $r = \rho\sin(\phi)$, and to find z and r, remember the $\phi - z - \rho$ triangle.
- Also memorize the $\rho^2\sin(\phi)$ term, although I won't ask you to derive it. To remember it, just think $\rho r = \rho\rho\sin(\phi) = \rho^2\sin(\phi)$
- I won't ask you to plot a point with given spherical coordinates, nor will I ask to change a point from spherical to rectangular coordinates (and vice-versa)
- Calculate integrals and volumes using spherical coordinates; problems 21, 22 − 28, and 30 in 15.8 are excellent practice problems.
- The problems in lecture are also great, and you can check out this YouTube video or this one.
- Remember that spherical coordinates are great for spheres and anything that involves $\sqrt{x^2 + y^2 + z^2}$.

- I could ask you to calculate the mass of the sun, although of course I will give you the density, see this video.

Section 15.9: Change of Variables

- There are two kinds of change of variables I could ask you:
- One where you choose u and v, like in $\int\int \cos\left(\frac{y-x}{y+x}\right) dxdy$. In that case, I will **NOT** give you u and v and you'll have to figure it out, either by using the function, or by using the region (like in the problem on the mock midterm). Problems $23 - 28$ in 15.9 are great sample problems, as well as the problems in lecture, the problem on the mock midterms/final and the actual midterm and the final from 2018 or this Jacobian 1 video
- Or one where x and y are in terms of u and v, like Problems 15 through 19. In that case, I will **give** you x and y. See Jacobian 2, and Change of Variables
- Don't forget about the absolute value!!!
- The easiest way to memorize the Jacobian definition is by using $dx = \left|\frac{dx}{du}\right| du$, or $du = \left|\frac{du}{dx}\right| dx$. Remember the motivating examples in lecture!
- Know how to rederive the Jacobian for polar coordinates
- Although unlikely, I could ask you to rederive the Jacobian in spherical coordinates
- I could ask you about Hyperbolic coordinates (see last problem on the midterm of 2018), but in that case, I would give you all the formulas for cosh and sinh . Check out: Hyperbolic Coordinates