MAT 267, Spring 2021, Test 3, Instructor: Peyam Tabrizian

Part I: Multiple Choice. Select the correct answer for each problem and make a table of the answers at the top of your first page for the exam. There are 7 problems, each worth 8 points, for a total of 56 possible points.

1. Evaluate the triple integral
$\int_0^1 \int_0^2 \int_0^3 \ 8xyz \ dxdydz$.

A. 10

B. 18

C. 24

D. 36

E. none of the above

2. Let E be the portion of the unit ball in the first octant and let $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$. Then $\iiint_E f(x, y, z) \ dV$ in spherical coordinates is

A. $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho \ d\rho \ d\phi \ d\theta$

B. $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^3 \sin\phi \ d\rho \ d\phi \ d\theta$

C. $\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^2 \sin^2\phi \ d\rho \ d\phi \ d\theta$

D. $\int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho \sin\phi \ d\rho \ d\phi \ d\theta$

E. none of the above

3. Let E be the cylinder with height h whose base is a disk of radius R in the xy-plane centered at the origin. If E has density $f(x, y, z) = x^2 + y^2 + z^2$, then what is the total mass of E, in terms of integrals?

A. $\int_0^{2\pi} \int_{-R}^{R} \int_0^h (r^2 + z^2) \ dz \ dr \ d\theta$

B. $\int_0^{2\pi} \int_0^{R} \int_0^h r^3 \ dz \ dr \ d\theta$

C. $\int_0^{2\pi} \int_0^{R} \int_0^h (r^3 + rz^2) \ dz \ dr \ d\theta$

D. $\int_0^{2\pi} \int_0^{R} \int_0^h r \ dz \ dr \ d\theta$

E. none of the above

4. Only one of the following vector fields is conservative. Which is it?

A. $\mathbf{F}(x, y) = \langle -y^2, x \rangle$

B. $\mathbf{F}(x, y) = \langle x^3 y, xy^3 + 1 \rangle$

C. $\mathbf{F}(x, y) = \langle 2y + 1, 3 \rangle$

D. $\mathbf{F}(x, y) = \langle x - 4y, y - 4x \rangle$

E. none of the above

5. Let C be the (curved) portion of the circle $x^2 + y^2 = 9$ in the first quadrant with counterclockwise orientation. Then $\int_C x \ ds$ is

A. 3

B. -9

C. 9

D. 0

E. none of the above

6. Let C be the unit circle with positive orientation and $\mathbf{F}(x, y) = \langle -2y^3, 2x^3 \rangle$. Using Green's Theorem, calculate the value of $\int_C \mathbf{F} \cdot d\mathbf{r}$.

A. 2π

B. $\frac{3\pi}{2}$

C. 3π

D. 0

E. none of the above

7. Which of the following is a potential function (antiderivative) for the conservative vector field $\mathbf{F}(x, y, z) = \langle 2x, 4y, 8z \rangle$?

A. $f(x, y, z) = x^2 + y^2 + z^2$

B. $f(x, y, z) = 2x^2 + 4y^2 + 8z^2$

C. $f(x, y, z) = 2x + 4y + 8z$

D. $f(x, y, z) = x^2 + 2y^2 + 4z^2$

E. none of the above

Part II: Free Response. Solve each problem and show all work clearly. Draw a box around your final answers and include units where applicable. Give exact answers to all questions; do not state the answers as decimal approximations. There are 3 problems worth a total of 44 points.

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1. [14 points] Use a triple integral in cylindrical coordinates to find the volume of the solid region E bounded by the cones $z = 6 - 2\sqrt{x^2 + y^2}$ and $z = \sqrt{x^2 + y^2}$. Make sure to draw a picture of E, labeling the two functions. You have to draw a picture and use triple integrals here.

2. [14 points] Find the work done of the force field $\mathbf{F}(x, y, z) = \langle -3y, 3x, 2z \rangle$ on an object that moves along the trajectory $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ from the point $(0, 0, 0)$ to the point $(2, 4, 8)$

3. [16 points] Define the vector field $\mathbf{F}(x, y) = \langle 4x + 3e^y, 3xe^y + 3y^2 \rangle$.

a) [3 pts] Check that $\mathbf{F}$ is a conservative vector field.

b) [10 pts] Find a potential function (antiderivative) for $\mathbf{F}$.

c) [3 pts] Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is any curve from $(3, 0)$ to $(1, 2)$. Simplify your answer.