MATH 2E − MIDTERM

Name: ____________________________

Student ID: ____________________________

Discussion Section time: (please circle)

1 − 2 PM   4 − 5 PM

Instructions: Welcome to your Midterm! You have 50 minutes to take this exam, for a total of 50 points. **You will lose 1 point if you don't completely fill out the information on this page.** No books, notes, calculators, or cellphones are allowed. Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. May your luck be spherical!!! :)

Academic Honesty Statement: I hereby certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating (no matter how small) results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to section 102.1 of the UCI Student Code of Conduct.

Signature: ____

1		10
2		10
3		10
4		10
5		10
Total		50

Spherical coordinates:

$$
\begin{aligned}
x =& \rho \sin(\phi)\cos(\theta) \\
y =& \rho \sin(\phi)\sin(\theta) \\
z =& \rho \cos(\phi) \\
\text{Jac} =& \rho^2 \sin(\phi)
\end{aligned}
$$

Date: Friday, February 7, 2020.

1. *(10 points)* Using the change of variables below, find the area of the region D bounded by the ellipse $x^2 - xy + y^2 = 8$.

$$\begin{cases} x = \left(\sqrt{2}\right) u - \left(\sqrt{\dfrac{2}{3}}\right) v \\ y = \left(\sqrt{2}\right) u + \left(\sqrt{\dfrac{2}{3}}\right) v \end{cases}$$

2. *(10 points)* Find $\int_C F \cdot dr$, where

$$F(x, y, z) = \langle e^x \sin(yz), ze^x \cos(yz), ye^x \cos(yz) \rangle$$

C is the line connecting $(1, 0, 0)$ and $(0, \pi, 1)$.

3. *(10 points)* Find the average value of the function

$$f(x, y, z) = 7\left(x^2 + y^2 + z^2\right)^2$$

over the solid E, where E is the region between the two surfaces $x^2+y^2+z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$ (Here $b > a > 0$). Include a picture of E.

Note: If you don't know how to calculate an average value, then just calculate the triple integral of f over E (for a maximum of 7 points)

4. *(10 points)* Calculate

$$\int\int\int_E 2\ dxdydz$$

E is the solid inside the surface $x^2 + z^2 = 1$ and between the surfaces $y = 2 - x^2 - z^2$ and $y = -1$. Include a picture of E.

5. *(10 points)* Derive (from scratch) the equations for spherical coordinates below. Include a picture.

$$\begin{cases} x = \rho\sin(\phi)\cos(\theta) \\ y = \rho\sin(\phi)\sin(\theta) \\ z = \rho\cos(\phi) \end{cases}$$