

## MATH S4062 – MIDTERM

Name	
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Signature	

**Instructions:** Welcome to your Midterm! You have 95 minutes to take this exam, for a total of 40 points. No books, notes, calculators, or cellphones are allowed. **Please write in complete sentences if you can.** Remember that you are not only graded on your final answer, but also on your work. If you need to continue your work on the back of the page, please clearly indicate so.

**Academic Honesty Statement:** With the signature above, I certify that the exam was taken by the person named and without any form of assistance and acknowledge that any form of cheating results in an automatic F in the course, and will be further subject to disciplinary consequences, pursuant to the Columbia University Honor Code.

1. (*10 points*) State and prove the Banach Fixed Point Theorem.<sup>1</sup>

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<sup>1</sup>You do not need to define the terms that you're using. Also, if you don't remember the statement, please ask me, and I can give it to you for 8 points out of 10

2. (10 points) Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given and let  $f_n(x) = f(nx)$ . Show that if  $f_n$  is equicontinuous on  $[-1, 1]$  then  $f$  must be constant on  $\mathbb{R}$ .

3. (10 points) Show the following series converges uniformly on  $[a, \infty)$  for all  $a > 0$  but does not converge uniformly on  $(0, \infty)$

$$\sum_{n=1}^{\infty} \frac{1}{1 + (n^2)x}$$

4. (10 points) Suppose  $f_n \in \mathcal{S}$  is a sequence of functions such that  $f_n \rightarrow f$  uniformly on  $\mathbb{R}$  and there is a constant  $M$  with

$$\lim_{n \rightarrow \infty} \int_{|y| > M} |f_n(y) - f(y)| dy = 0$$

Show directly that  $\widehat{f}_n \rightarrow \widehat{f}$  uniformly on  $\mathbb{R}$

**Note:** If  $g \in \mathcal{S}$  then  $\widehat{g}(x) = \int_{-\infty}^{\infty} g(y) e^{-2\pi ixy} dy$